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Financial Management and Accounting

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VOLUME 3

Investment Analysis and Portfolio Management

FINANCIAL MANAGEMENT
AND ACCOUNTING

Investment Analysis and Portfolio Management

Note

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CONTENTS

PREFACE

The purpose of this book is to provide a good understanding of security analysis and portfolio management. Security analysis is a component of the investment process that involves determining the prospective future benefits of a security, the conditions under which such benefits will be received, and the likelihood that such conditions will occur. The portfolio management is the task of planning, implementing and overseeing the funds of an individual or an institutional investor, depending on their preferences and needs. From the above it follows that this book is concerned with the characteristics and analysis of individual securities, as well as with the theory and practice of optimally combining securities into portfolios.

An investment management book should tie together theory and practice. Investing in a theoretical world is rather simple; however, investing in a world with transaction costs and strict regulations means that theory must be adapted to real world circumstances. Considerable effort has been made in creating a textbook in which theory and practice can be harmonically combined into a conceptually easy-to-understand script. To serve this purpose, it was decided that several examples and activities should follow each theoretical issue. Furthermore, the text is quantitative enough to support analytical rigor, but advanced topics are generally presented without overly mathematical treatment.

The theory of investment was initially developed about 50 years ago. However only in the last 10 - 15 years have the concepts begun to be put into practice, mainly because major theoretical breakthroughs have simplified the amount and type of inputs to the portfolio problem. The main advantage of the simplifications brought forward by these theoretical breakthroughs was that the portfolio selection process and the final portfolios selected have a structure with a clear economic rationale, so that both the practicing security analyst and the economist can relate.

Investment analysis and portfolio management theory continues to evolve at a fast pace. However, as the reader will see, empirical tests suggest so far that the modern theory, as it stands up to now, can provide great insight into the process of shaping and managing a portfolio. This happens because the main principles of investment theory still remain very important. Nevertheless, a great deal of innovative work in broadening the application of theory is being done by researchers, analysts and portfolio managers at investment firms.

After reading this book, the student will have realized the notion of the risk-return trade-off when planning an investor’s asset allocation procedure in a portfolio. He/she will be able to form a portfolio based on objective assumptions and to evaluate its performance. He/she will possess adequate knowledge about the money market and the capital market and he/she will be able to value a fixed-income security. The reader will be able to tell which stock is overpriced or undervalued and assess whether a market can be considered as an efficient market. Summing up, it can be said that the main objective of the book is to develop a way of analyzing and thinking about investments.

This book is divided into ten chapters, which cover the most important subjects of investments. The structure of the book is as follows: The first chapter presents a short introduction to the investment management process. The second chapter discusses the returns and risks related to investments. The third chapter examines both money and capital markets. The fourth chapter explains the valuation of fixed income securities. The fifth chapter illustrates stock valuation. The sixth chapter describes the portfolio theory and the seventh draws its attention to the capital market theory. The eighth chapter covers the evaluation of portfolio performance and the ninth chapter explores the efficient markets. Finally, the tenth chapter takes a look at technical analysis.

Each chapter includes the scope of the chapter, the learning objectives, some key words, an introductory preview, and a synopsis. Additionally, in most of the chapters, as was already mentioned, there are a number of examples with their answers, and various activities, whose answers are presented at the end of the chapter. Please note that the use of “he”, “his” and so on, is only for the purpose of simplifying the exposition; it should be understood as including both sexes.

CHAPTER 1

INTRODUCTION

The Scope of the Chapter

In this chapter, we help you understand the investment management process and the role played by the financial system.

Learning Objectives

When you have finished studying this chapter, you will be able to:
- describe the two parts of the security investment process
- distinguish between direct finance and indirect finance
- explain the difference between the primary market and the secondary market.

Key Words

- Investment
- Portfolio management
- Security analysis
- Direct and indirect financing
- Primary and secondary market

Introductory Comments

An investment is a commitment of funds made for a period of time in order to derive a positive rate of return in the future. Investment differs from speculation in the following two points: First, the investor usually has a relatively long time horizon over which he expects the returns of his invested funds to be realized, while the speculator usually has a short time horizon. Second, the investor expects a normal rate of return from his investment earned on a rather consistent basis, whereas the speculator expects an extremely high rate of return without being interested in consistency over time. The same stock can be purchased either as a speculation or an investment, depending on the motivation of the purchaser. Securities may serve both for investment or speculative aims, depending on the expected rate of return and the time horizon of the investor (or speculator). This book deals with investments, although speculative situations will also be considered.

This chapter includes four sections. The first section presents an overview of the investment process. The second section describes the financial system's general characteristics. The third section examines the flow of funds from lenders to borrowers. Finally, the fourth section presents the structure of the book.

1.1 THE INVESTMENT PROCESS

An **investment** can be defined as a commitment of funds for a period of time expected to yield additional funds to the investor[1]. Each investment requires that the investor avoid certain consumption today, for an uncertain future reward. Thus, every investment contains risk to some extent. From the above it follows that investments involve the management of the investors' wealth, which is composed from their current income and the present value of all their future income.

Investments are most often made on securities. The security investment process can be divided in two parts: **security analysis** and **portfolio management**. Security analysis is defined as the process of determining whether a security has been priced appropriately by the investors in the market. In other words, security analysis is looking for underpriced securities. The traditional security analysis approach gives emphasis to the estimated future cash inflows that crop up from the securities, which are discounted back to the present. This present value is then compared with the security's current market price. If the current market price is below (above) the present value, a purchase (sale) is recommended. The modern security analysis approach focuses not only in the estimation of the securities' returns, but also in the risk that the securities entail.

Portfolio is the combination of various assets that an investor holds. These assets may constitute the entire wealth of the investor or a part of it. **Portfolio management** is the procedure of combining several securities in a portfolio, depending on the investor's individual preferences and needs, to continuously monitor this portfolio and to evaluate its performance. The traditional approach of portfolio management emphasizes in the selection of the securities that best fit the investor's individual preferences and needs. For example, the financial counselor of a young single investor would advise him/her to purchase stocks of new and dynamic firms which are growing rapidly. On the contrary, the financial counselor of a retired investor would advise him/her to buy government bonds and stocks of big, established and stable companies (e.g. utilities). The modern portfolio management approach is based on the estimates of return and risk of the portfolio along with the investor's preferences between return and risk. This approach takes into account that the risk of a portfolio may differ from the sum of the risks of the individual stocks that are included in the portfolio. This book intends to cover succinctly the both parts of the investment process; the security analysis and the portfolio management.

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[1] *The definition that dictionaries usually provide for the investment is the use of money for the purchase of anything that is expected to return interest or profit.*

1.2 THE FINANCIAL SYSTEM

All the economic units can be comprised in one of the following three major categories: households, business firms, and the public sector. During a given period of time, each of them has certain receipts of income (inflows) and expenditures (outflows). According to the net result (inflows minus outflows) at the end of the time period, each unit must fall into one of the three groups:

- a surplus-budget unit,
- a deficit-budget unit, or
- a balanced-budget unit.

The surplus-budget units seek to invest their surplus capital (net lenders of funds), whereas the deficit-budget units look for additional capital in order to finance their deficits (net borrowers of funds). The balanced-budget units are neither net lenders nor net borrowers. Thus, these different categories have supplementary objectives and look for a mechanism that it will bring them in contact and facilitate their transactions. This is accomplished by the financial system. The **financial system** deals with the transfer of scarce funds from those who save and lend (surplus-budget units) to those who wish to borrow and invest (deficit-budget units), and the acceptance of the ultimate lenders' legal representation of the right to receive prospective future benefits, under stated conditions, from the ultimate borrowers. These claims represent financial assets such as stocks, treasury bills or bonds. More specifically, an asset is any possession that has value in an exchange. Assets can be either tangible or intangible. A tangible, or real asset, is one whose value depends on particular physical properties (e.g. land, buildings, machines etc.). An intangible asset represents legal claims to some future benefit. Financial assets (also called financial instruments, financial investments or securities) are intangible assets, the typical future benefit is a claim to future cash. This book deals with investments in financial assets.

1.3 VARIOUS WAYS OF FINANCING

The transfer of funds from ultimate lenders to ultimate borrowers can be accomplished in the following two ways[2]:

- **Direct finance or invest**. In this case, a borrower and a lender communicate directly and exchange funds in return for financial assets without the aid of a third party. The purchase of stocks or bonds directly from the company issuing them is an example of direct finance. The claims arising from direct finance are usually called **primary securities** because they flow directly from the ultimate borrower to the ultimate lender of funds. Direct finance or investing has a disadvantage: there must be a coincidence of wants between the borrower and the lender in terms of the amount and form of a financial transaction. Without that fundamental coincidence, direct finance breaks down.
- **Indirect finance or investing**. In this case, the financial transactions (especially the borrowing and lending of money) are carried out with the help of financial intermediaries. Financial intermediaries or financial institutions issue financial claims against themselves (meaning that they sell financial assets representing claims on themselves in return for cash) – often called **secondary securities** – and use the proceeds from this issuance to purchase primarily the financial assets of others. Examples of financial intermediaries are commercial banks, insurance companies, mutual funds, finance companies, and similar organizations. Nowadays, most investments are indirect.

A financial market is an organized institutional structure that provides a mechanism for creating and exchanging financial assets. The financial market is the heart of the financial system, attracting and allocating savings and setting interest rates and the prices of financial assets (stocks, bonds, etc.).

There are many ways to classify financial markets. One way is by the maturity of the claims. In this case there are the following two categories:

- **Money market**, where short-term financial assets are traded (with maturity of one year or less).
- **Capital market**, where long-term financial assets are traded (with maturity of more than one year).

Another way to classify financial markets is based on whether the financial claims are newly issued or not. In this case there are the following two categories:

- **Primary market**, where newly issued financial assets are traded.
- **Secondary market**, where financial assets are traded after they are initially offered in the primary market.

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[2] *Some authors propose a third way, called semi direct finance, which includes any financial transaction that is assisted by a security broker or dealer.*

Synopsis

- **An investment can be defined as a commitment of funds for a period of time expected to yield additional funds to the investor. The security investment process can be divided in two parts: security analysis and portfolio management. Security analysis is defined as the process of determining whether a security has been priced appropriately by the investors in the market. In other words, security analysis is looking for underpriced securities. Portfolio management is the procedure of combining several securities in a portfolio, depending on the investor's individual preferences and needs, to continuously monitor this portfolio and to evaluate its performance.**
- **The financial system deals with the transfer of scarce funds from those who save and lend (surplus-budget units) to those who wish to borrow and invest (deficit-budget units), and the acceptance of the ultimate lenders' legal representation of the right to receive prospective future benefits, under stated conditions, from the ultimate borrowers. The transfer of funds from ultimate lenders to ultimate borrowers can be accomplished either by direct finance or by indirect finance. In direct finance, a borrower and a lender communicate directly and exchange funds in return for financial assets without the aid of a third party. In indirect finance, the financial transactions (especially the borrowing and lending of money) are carried on through a financial intermediary.**
- **A financial market is an organized institutional structure that provides a mechanism for creating and exchanging financial assets. Financial markets can be classified as money markets and capital markets, or as primary markets and secondary markets.**

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CHAPTER 2

RETURN AND RISK

The Scope of the Chapter

In this chapter, we help you understand some basic concepts regarding the investments' return and risk, and explain how they can be measured.

Learning Objectives

When you have finished studying this chapter, you will be able to:

- distinguish between the three different concepts of return on an investment
- define the two components of the rate of return on an investment
- mention the most important risk sources on an investment
- explain the difference between systematic and unsystematic risk
- measure the rate of return and risk on an investment
- estimate (i.e. forecast) the expected rate of return and risk on an investment that you intend to pursue.

Key Words

- Realized return
- Expected return
- Required return
- Yield
- Capital gain or loss
- Risk
- Interest rate risk
- Inflation risk or purchasing power risk
- Market risk
- Business risk
- Financial risk
- Liquidity risk
- Exchange rate (or currency) risk
- Country (or political) risk
- Holding period return

Introductory Comments

In the following chapters we will deal with the analysis and evaluation of fixed income securities and shares. In order to evaluate these securities, it is essential that you have comprehended the basics of return and risk. Moreover, you should know how an investor measures the rate of return and the risk on an investment which he/she has already undertaken or intends to undertake.

This chapter contains three major sections. In the first section the basics concepts of return and risk are examined. The next section presents how an investor measures the rate of return and the risk on an investment that has already been carried out. Finally, the third section describes how an investor can estimate the rate of return and the risk on an investment that has not been realized yet.

2.1 RETURN

It is very important for the investor to understand the concept of return on an investment because it measures the change in his/her wealth resulting from this investment. The term "return", however, can be misleading since no single measure of return can answer all possible questions regarding the above result. For this reason it is important to distinguish between realized return, expected return and required return.

Realized return (or ex post return or historical return) is the actual rate of return on an investment over some specific time period. If, for example, an investor deposits €1,000 in a bank account at the beginning of the year and the balance of this account is €1,100 at the end of the year, the realized return over this period is $\left(\frac{1,100-1,000}{1,000}\right)=10\%$.

Expected return (or ex ante return or anticipated return) is the return that an investor anticipates receiving in the future from an investment over a holding time period. Since the future is uncertain, the expected return may be realized or may not be realized. Thus, the realized return on an investment may differ from the expected return.

Required return is the minimum rate of return that the investors require from an investment in order to undertake it. Put differently, the required return is the minimum rate of return they should accept from an investment to compensate them for deferring consumption. The required return has three components:

- First, the real risk-free rate of return, which is the compensation that an investor requires in order to postpone his/her current consumption (pure time value of money). This is the basic interest rate, assuming no inflation and no uncertainty about future flows, and is based upon the long run real growth rate of the economy.
- Second, the expected rate of inflation.
- Third, a risk premium (i.e. a compensation that investors demand for an investment's uncertainty), which depends on various sources of uncertainty such as business risk, financial risk, liquidity risk, exchange rate risk and country (political) risk.

2.2 YIELD AND CAPITAL GAINS OR LOSSES

When an investor undertakes an investment, he/she forgoes current consumption in order to increase future consumption. Thus, what the investor really cares for is the increase of his/her wealth from the investment. This change in the investor's wealth may result from the receipt of current income, such as dividends or interest, and/or from the change in the price of the investment. Therefore, an investment's return consists of two parts:

- **Yield**, which is the periodic cash inflows from an investment. In the case of stocks, this income has the form of dividends, while in the case of bonds it has the form of coupons. These returns are usually expressed as either a percentage of the purchase price of the security or as a percentage of the current stock price.
- **Capital gain or loss,** which is the price change of the security over a time period. If, for example, a share is bought for €1,000 and is sold (or can be sold) for €1,200, the difference between the two prices (i.e. the €200) is capital gain. If a share is bought for €1,000 and is sold (or can be sold) for €800, the difference between the two prices (i.e. €200) is loss.

  From the above it follows that:

		+ Capital gain
Security's total return =	Yield	or
		- Capital loss

2.3 RISK

The definition that the dictionaries usually provide for risk[1] is the probability of loss, damage or injury. However, this definition does not quantify the concept of risk with precision. For this reason, risk could be defined as the probability the actual outcome of an investment differs from the expected[2]. Investors, however, do not view possible returns above the expected return as an unfavorable outcome. In fact, such outcomes are quite favorable. Risk measures based on below – the – expected value variability are difficult to work with, however, and are moreover unnecessary as long as the distribution of future returns is reasonably symmetric about the expected value. Needless to say, the empirical evidence suggests that the historical distribution of returns on stocks is approximately symmetrical[3].

Generally, the more the possible outcomes of an investment, the higher is its risk. In case that there is no variability of the possible outcomes around the expected value, there is no risk. For example, suppose that an investor buys an annual treasury bill that yields 10% and the investor holds it to maturity. In this case, the treasury bill bears no risk, since in its expiry it will offer to its holder the expected return of 10%. From the above it follows that **risk is the variability of the possible outcomes around their expected value or their mean.**

[1] *There is an important distinction between risk and uncertainty in the economic theory based on Knight (1921). The risk refers to cases where the outcome is uncertain, but the probabilities of alternative possible outcomes are known or can be precisely estimated (through empirical experiments or through the use of statistical data). The uncertainty refers to situations where the agent cannot (or does not) assign actual probabilities to the alternative possible occurrences. The reader should note that the introduction of subjective probability has greatly diminished the significance of the distinction between risk and uncertainty. By assigning subjective probabilities to decision problems, an inherently uncertain situation can be transformed into a risky choice.*

[2] *The precise definition of risk is given later in this chapter.*

[3] *See chapters 1 and 2 of Fama (1976).*

2.4 SOURCES OF RISK

All investments bear risk that makes their future returns uncertain. The total risk of an investment derives from various sources, the major of which are the following:

Interest rate risk is the possible variability of returns on an investment, caused by fluctuations in the interest rates of the markets. Assuming that all others remain constant, a change in interest rates will result in an opposite change in the prices of the securities. If, for example, the market interest rates rise, then the values of bonds, shares and other investments will fall. The opposite will happen if interest rates fall. The inverse relation between interest rates and security prices is due to the securities' valuation procedure. We will deal with this subject in a following chapter.

Inflation risk or purchasing power risk is the possible variability of returns on an investment, due to the reduction of the purchasing power of the invested capital. To the extent that the future inflation[4] is uncertain, the real return (i.e. the deflated return) bears risk even if its nominal return is certain (as, for example, in the case of the treasury bills return).

Market risk is the possible variability of returns on an investment, caused by changes in the entire stock market. The causes of this variability are varied, but it is mainly due to a change in investor's attitudes toward securities that stem from tangible (e.g. expectations of lower corporate earnings) and intangible events. Intangible events are related to market psychology (e.g. overreactions to lower expected corporate earnings and the resultant panic selling). Stock market fluctuations affect all investments, but equities are the investments that are more exposed to market risk.

Business risk is the possible variability of returns on an investment in a firm, caused by the nature of a firm's business. For example, investing in firm A, which sells foods and experiences stable sales and earnings growth over time, has lower business risk than investing in firm B, which sells cars and experiences significant fluctuations in its sales and earnings over time.

Financial risk is the possible variability of returns on an investment in a firm, caused by the way in which the firm finances its activities. The more debt a firm uses, the more it is exposed to financial risk.

Liquidity risk is the risk introduced by the secondary market for the investment. When an investor buys a security, he/she expects to be able to convert the security

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[4] *Inflation is a process of continuously rising prices, or equivalently, of continuously falling value of money. The most commonly used measures of inflation are the percentage rate of change in a country's Consumer Price Index or in its Gross National Product deflator.*

into cash whenever the need arises. Liquidity is the ability to buy or sell an asset quickly and at a reasonable price. Thus, the higher the uncertainty regarding the time required for, and the price to be received at, the conversion of an asset to cash, the higher the liquidity risk.

Exchange rate risk or currency risk is the uncertainty which the investor faces when he/she has acquired securities denominated in a foreign currency, and the securities' returns are converted from the foreign currency to his/her own. If, for example, an American investor buys stocks in Athens Stock Exchange, his/her returns will also depend on the movement of the exchange rate between dollar and euro.

Country risk or political risk is the uncertainty of returns, caused by the possibility of important changes in the political or economic environment of a country.

These are the main sources of investment risk that are taken under consideration by the traditional approach. Modern investment analysis separates risk into two categories: to those risks related to the total market movements and to those that each individual investment has. In the modern terminology, these two risks are mentioned as the **systematic or market risk** and the **unsystematic risk** respectively. Thus, in modern investment analysis the total risk of an investment is measured as follows:

$$\text{Total risk} = \text{Systematic risk} + \text{Unsystematic risk}$$

The systematic risk (or market risk or nondiversifiable risk) is the investment risk which is related to movements of the entire market and cannot be eliminated by portfolio diversification[5]. This risk is due to general market and economic conditions that cannot be diversified away, and are independent from each individual asset contained in the investor's portfolio. For example, in 19 October 1987 there was a significant fall of all share prices in the New York Stock Exchange. That day the Dow Jones Industrial Average index fell by 22%. All securities, either shares or bonds, have systematic risk, as this risk includes the interest-rate risk, the market risk and the inflation risk. On the other hand, the unsystematic risk (or unique risk, or residual risk, or company-specific risk, or diversifiable risk) is the risk which is unique to an asset derived from its particular characteristics, and consequently, can be eliminated with portfolio diversification. Specifically, the unique risk of any asset is offset by the unique variability of the other assets in the portfolio. Strikes, the outcome of an unfavorable litigation, or a natural catastrophe are examples of this type of risk. Although most securities include unsystematic risk to some extent, this risk is mainly related to stocks. The unsystematic risk includes the business risk, the financial risk and the liquidity risk[6].

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[5] *Diversification is the undertaking of various investments with the aim of risk minimization. Thus, an investor who wishes to diversify his/her portfolio will buy securities of different types. In the case of a firm, diversification involves the production of different products, so that any difficulties in the sale of a specific product can be surpassed or compensated by the successful sales of the other products.*

[6] *The liquidity risk may also be included in the category of systematic risk if the entire market has lack of liquidity during the examined period.*

2.5 MEASURES OF RETURN AND RISK[7]

2.5.1 Return

There are various ways to measure the realized return of an investment. The most common methods can be described in the following example. Suppose that an investor invested €1,000,000 at 1/1/2000 and liquidated the investment at 31/12/2001 receiving €1,100,000[8]. The two-year period during which the investor owns the investment is called its **holding period**, and the return for that period is the **holding period return** (HPR). The holding period return is calculated as follows:

$$\text{HPR} = \frac{\text{Ending value of investment}}{\text{Beginning value of investment}}$$

The ending value of investment includes the security current value in the market plus the cash inflows (e.g. dividends, coupons, etc.) that the holder has received during the holding period. Thus, the holding period return in the above example is HPR 1,100,000/1,000,000 = 1.10. This value is always greater than or equal to zero and never negative. A value greater than one represents a positive investment return, while a value less than one indicates a negative return. An HPR of zero means that the investor lost all his/her money. The above return can be also expressed in percentage terms on an annual basis. The percentage return, referred to as the **holding period yield (HPY)**, is calculated as follows:

$$\text{HPY} = \text{HPR} - 1.00$$

In our example the holding period yield is HPY = 1.10 – 1.00 = 0.10 or 10%. To derive an annual HPY, when the holding period is different from one year, we first calculate the annual HPR and then subtract 1.00. The calculation of annual HPR is:

$$\text{Annual HPR} = \text{HPR}^{1/n}$$

where n is the number of years that the investment is held.

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[7] This section considers historical measures of return and risk. We turn to estimating expected rates of return and risk later in this chapter.

[8] The additional €100,000 may come from yield (e.g. dividends), from capital gains, or both.

In our example:

$$HPR = 1{,}100{,}000/1{,}000{,}000 = 1.10$$

$$HPY = 1.10 - 1.00 = 0.10 \text{ or } 10\%$$

$$\text{Annual HPR} = 1.10^{1/2} = 1.0488$$

$$\text{Annual HPY} = 1.0488 - 1.00 = 0.0488 \text{ or } 4.88\%.$$

At this point we should note that the ending value of the investment includes all the cash inflows that the investor received during the holding period, as well as the possible capital gains.

Activity 1/Chapter 2

On January 15, you bought shares of stock for €25 a share and a year later you sold them for €26.5 a share. During the year you received a cash dividend of €0.5 a share. Compute the holding period return (HPR) and the holding period yield (HPY) on your investment.

The annual holding period yield is useful when an investor wants to measure the return of a single investment for a single year. Many times, however, an investor wants to calculate the average return of an investment which he/she has held for several years. During these years, the annual returns were sometimes high, sometimes low and sometimes negative. In this instance, the investor may want a summary figure that indicates the performance of this investment over the whole time period. Alternatively, an investor may wish to calculate the mean return of a portfolio of investments for a single year or for a number of years. In these cases, the investor may use one of the following statistical measures:

(a) Arithmetic mean (AM). It is the sum of annual HPY divided by the number of years (n) that the investment is held:

$$AM = \sum HPY / n$$

(b) Geometric mean (GM). It is the nth root of the product of the HPRs for n years:

$$GM = \pi^{1/n} - 1$$

where π is the product of the annual holding period returns (HPR), and has the following form:

$$\pi = (HPR_1) \times (HPR_2) \times (HPR_3) \times \ldots \times (HPR_n)$$

The geometric mean measures the compounded rate of return on an investment during a holding period, assuming that all cash inflows are reinvested. Consequently, when we are interested in the long-term mean rate of return on an investment, the geometric mean shows the actual average change in the investor's wealth.

Example 1

During the last three years you own an investment with the following data. Compute the arithmetic mean rate of return, the geometric mean return and discuss their difference.

Year	Beginning value	Ending value	HPR	HPY
1	1,000	1,200	1.20	0.20
2	1,200	1,320	1.10	0.10
3	1,320	1,188	0.90	-0.10

Answer:

$$AM = [(0.20)+(0.10)+(-0.10)]/3 = 0.20/3 = 0.0667 \text{ or } 6.67\%$$

$$GM = [(1.20)\times(1.10)\times(0.90)]^{1/3} - 1 = (1.188)^{1/3} - 1 = 1.0591 - 1 = 0.0591 \text{ or } 5.91\%$$

The annual average return of the investor is 6.67%. This rate of return takes into account all returns, either high (as in year 1), or low (as in year 3). Therefore, the best estimation for the next year's mean return (i.e. year 4) will be 6.67%. On the other hand, the actual annual return that the investor enjoyed during the examined period (3 year period), was 5.91%. This can be explained as follows: €1,000 compounded at an annual interest rate of 5.91% for 3 years provide an ending value, at the end of the third year, equal to €1.188[9].

2.5.2 Arithmetic mean vs. geometric mean

When should we use the arithmetic mean and when the geometric mean? The arithmetic mean should be used when we want to compute the average investment return for a single period or year (for example, for 2004). On the contrary, the geometric mean should be used in order to compute the average return on an investment over multiple periods (for example, during the period 1994-2004). In case we attempt to measure the return on an investment over multiple periods by employing the arithmetic mean, we may receive larger results than those true. In general, when the rates of returns are the same for all years, the geometric and the arithmetic means will give the same result. When the rates of returns change from year to year, the geometric mean is lower than the arithmetic; this happens because the geometric mean reflects the return's volatility. The higher the return's volatility, the higher the difference between the two means.

[9] *$TV = 1{,}000\times(1+0.0591)^3 = 1{,}188$.*

Example 2

Consider a security that increases in price from €1,000 to €1,500 during the first year and drops back to €1,000 during the second year. In this case there is no change in wealth from this investment over the two-year period. Compute the arithmetic mean rate of return and the geometric mean rate of return. Which of the two means measures accurately the change in wealth from this investment over the two-year period?

Answer:

Year	Beginning value	Ending value	HPR	HPY
1	1,000	1,500	1.50	0.50
2	1,500	1,000	0.67	-0.33

$$AM = [(0.50)+(-0.33)]/2 = 0.17/2 = 0.085 \text{ or } 8.5\%$$

$$GM = [(1.50)\times(0.67)]^{1/2} - 1 = (1.00)^{1/2} - 1 = 1.00 - 1 = 0 \text{ or } 0\%$$

This investment did not change the investor's wealth, and consequently, the geometric mean is the proper measure (0%), while the arithmetic mean provides wrong results (8.5%).

2.5.3 Risk

We have defined risk as the **variability of possible outcomes around their expected value or their mean.** One of the most popular statistical measures of the variation of a random variable around its mean or expected value is the variance. The problem with using the variance as a measure of dispersion is that variance is expressed in terms of squared units of the random variable (in our case, returns). The square root of the variance, called standard deviation, is a more meaningful measure of the degree of dispersion. Consequently, the standard deviation measures the total risk of an asset or a portfolio[10] and it is calculated by using historical data as follows[11]:

$$\sigma = \left[\frac{\sum_{i=1}^{n} \left(X_i - \overline{X} \right)^2}{(n-1)} \right]^{\frac{1}{2}}$$

where σ = the standard deviation of the series, X_i = observation i on variable X, which is equal to the HPY, $\overline{X}$ = the average sample value for variable X, and n = the number of observations in the sample.

[10] *The variance and the standard deviation are equally acceptable measures of the total risk of an investment.*

[11] *This formula gives the standard deviation of the sample. The standard deviation of the population is found if we divide by n instead of n -1.*

Example 3

Use the data from example 1 and compute the risk of the investment.
Answer:

Year	HPY	HPY-AM	(HPY-AM)^2
1	0.20	0.1333	0.0178
2	0.10	0.0333	0.0011
3	-0.10	-0.1667	0.0278
	AM = 0.0667		Sum = 0.0467

$$AM = [(0.20)+(0.10)+(-0.10)]/3 = 0.20/3 = 0.0667 \text{ or } 6.67\%$$

$$\sum (X - \bar{X})^2 = \sum (HPY - AM)^2 = 0.0467$$

$$\sigma = [\ 0.0467/2\ ]^{1/2} = 0.1528 \text{ or } 15.28\%.$$

This example demonstrates that when we know historical rates of return on an investment during a specific time period, we can compute the total investment risk during this period. In the above case the investor's average annual rate of return was 6.67%, and the standard deviation of annual rates of return was 15.28%. This information is useful for the evaluation of a portfolio past performance, and the prediction of the total risk that this portfolio will bear in the future.

2.6 ESTIMATES OF RETURN AND RISK

2.6.1 Return

An investor that plans to undertake an investment expects or anticipates a certain rate of return. Since the future is uncertain, this return may be or may not be realized. To moderate this uncertainty, the investor should determine how certain the expected rate of return on an investment is, by analyzing estimates of expected returns. To do so, he/she assigns probability values to all possible returns. These probabilities range from zero, which indicates that the return is not possible to occur, to one, which means that the specified return will certainly occur. The probability for each return to occur increases as the return becomes more certain. Note that the sum of all probabilities is one. These probabilities are typically subjective estimates based on the past performance of the investment, and on the investor's expectations for the future. A **probability distribution** of returns is a function that describes all the possible returns on an investment and the probabilities of those values occurring. A probability distribution of returns is characterized by a number of measures. The two most often used measures are the expected value (or in this case the expected rate of return) and the variance (or standard deviation).

Expected rate of return is the weighted average of all the investment's possible returns, where the weights are the probabilities associated with the returns. That is:

$$\mathrm{E}(r) = \sum_{i=1}^{n} P_i r_i$$

where, E(r) = the expected return from an investment, r_i = the i^{th} possible return, P_i = the probability of the i^{th} possible return occurring, and n = the number of possible returns.

Example 4

You consider an investment and you believe that there is a 50% probability the investment to provide a 15% rate of return, a 30% probability to provide a 12% rate of return, and a 20% probability to provide a 7% rate of return. What is the expected rate of return from this investment?

Answer: The expected rate of return is calculated as the sum of the products of each of the possible returns times their associated probabilities. That is:

E(r) = (0.50×0.15) + (0.30×0.12) + (0.20×0.07) = 0.1250 or 12.5%.

2.6.2 Absolute measures of risk

The expected value lies at the center of the distribution of returns. Most of the possible returns lie either above or below it. The dispersion of possible returns about the expected return is measured by the variance (and equivalently, the standard deviation) of the probability distribution and can be used as a proxy of risk. The variance of returns is calculated as the weighted average of the squared deviations from the expected rate of return, where the weights are the probabilities associated with the returns. The standard deviation of returns can be calculated as follows:

$$\sigma = \left\{ \sum_{i=1}^{n} P_i \left[r_i - \mathrm{E}(r) \right]^2 \right\}^{\frac{1}{2}}$$

where, σ = the standard deviation of returns, r_i = the i[th] possible return, P_i = the probability of the i[th] possible return occurring, E(r) = the expected return from an investment, and n = the number of possible returns.

The variance (Var) is:

$$\mathrm{Var} = \sigma^2$$

The variance (or the standard deviation) is thus a quantitative description of risk. Moreover, this measure of risk is simply a proxy or surrogate for risk, since other measures could be used. The problem with using the variance as a measure of dispersion is that variance is expressed in terms of squared units of returns. This is not understood by most people. The standard deviation overcomes this problem by simply taking the square root of the variance, thus producing a statistic expressed in the same units as the expected value.

Note that the standard deviations of well diversified portfolios[12] remain relatively stable over time. Consequently, standard deviations calculated by using past returns can be considered as representative of standard deviations that will occur in the future. However, in case of individual securities, this kind of approach is particularly dangerous, as the standard deviations that have been computed with past data may considerably differ from those that will be realized in the future.

[12] *A well diversified portfolio is one that contains predominately systematic risk, because most of its unsystematic risk has been eliminated. A perfectly diversified portfolio is one that its risk is a reflection of market risk (that is, all of its unsystematic risk has been eliminated).*

Example 5

Use the data from example 4 and compute the risk of the investment.
Answer:

(P)	(HPY or r)	(P)×(r)	HPY-E(r)	$[HPY-E(r)]^2$	$(P)\times[HPY-E(r)]^2$
0.50	0.15	0.075	0.025	0.00060	0.00030
0.30	0.12	0.036	-0.005	0.00003	0.00001
0.20	0.07	0.014	-0.055	0.00300	0.00060
		E(r) = 0.1250			Sum = 0.00091

Expected return: $[E(r)] = (0.50\times0.15) + (0.30\times0.12)+(0.20\times0.07) = 0.1250$ or 12.5%.
Variance: $(\sigma^2) = \Sigma\{(P)\times[HPY - E(r)]\}^2 = 0.00091$
Standard deviation: $(\sigma) = \{\Sigma(P)\times[HPY - E(r)]^2\}^{1/2} = 0.00091^{1/2} = 0.0302$ or 3.02%

Therefore, the investor's expected return is 12.5% and the standard deviation of the investor's expectations (i.e. the risk) is 3.02%.

2.6.3 A relative measure of risk

Sometimes financial analysts or investors wish to compare the risk of investments that have major differences in their expected returns. In this case, the use of variance or standard deviation can lead to wrong conclusions. This is because variance and standard deviation are absolute measures of variation. In this case, we need a relative measure of variation, the coefficient of variation (CV). **The coefficient of variation measures the risk per unit of expected return and is calculated as the ratio of the standard deviation divided by the expected return.** It may be computed as follows:

$$CV = \frac{\sigma}{E(r)}$$

Higher values of the coefficient of variation indicate higher variability per unit of expected return and consequently higher relative risk. This measure of relative variability and risk is used by investors to compare alternative investments with widely different rates of return and standard deviations of returns.

Example 6

The expected returns and the standard deviations of investments A and B are presented in the following table. Compare the risk of these two investments.

	Investment A	Investment B
Expected return	0.08	0.14
Standard deviation	0.05	0.07

Answer:

Comparing absolute measures of risk, investment B bears more risk than A, because the standard deviation of B (σ_B = 0.07) is higher than that of A (σ_A = 0.05). However, the relative dispersion of investment B is smaller than the relative dispersion of A.

$$CV_A = 0.05/0.08 = 0.625$$

$$CV_B = 0.07/0.14 = 0.500$$

Therefore, the coefficient of variation shows that investment B contains less risk per unit of expected return than investment A, and consequently investment A is riskier than investment B.

Activity 2/Chapter 2

During the past five years, you owned two stocks that had the following annual rates of return:

Year	Stock A (HPY_A)	Stock B (HPY_B)
1	0.20	0.09
2	0.10	0.05
3	-0.14	-0.11
4	-0.05	0.04
5	0.17	0.06

(a) Compute the arithmetic mean annual rate of return for each stock. Which stock is most desirable by this measure?

(b) Compute the standard deviation of the annual rate of return for each stock. Which stock is preferable, by this absolute measure of risk?

(c) Compute the coefficient of variation for each stock. Which stock is preferable, by this relative measure of risk?

(d) Compute the geometric mean annual rate of return for each stock. Which stock is most desirable by this measure? Discuss the difference between the arithmetic mean return and the geometric mean return for each stock. Relate the differences in the mean returns to the standard deviation of the return for each stock.

2.6.4 Relationship between return and risk

Theoretical and empirical research suggests that there is a positive relationship between return and risk. This happens because investors are risk averse. Thus, investments which bear higher risk should provide the investors with higher rates of return to compensate them for the additional risk they undertake. Therefore, the higher the risk an investor undertakes, the higher the return the investor expects to receive from this investment. Actually, the above expectations of the investor are usually confirmed when the time horizon of the investment is long enough (for example 20 years). On the contrary, if the investment's time horizon is very short, (for example 1 or 2 years), the positive relationship between return and risk may not appear.

Activity 3/Chapter 2

An investor plans to buy stocks and has to choose between the stock A and the stock B. He/she expects that the possible returns of stocks A and B and their corresponding probabilities will be the following:

Probabilities of A (P_A)	Possible returns of A (HPY_A or r_A)	Probabilities of B (P_B)	Possible returns of B (HPY_B or r_B)
0.20	-0.15	0.15	-0.50
0.15	0.00	0.15	-0.30
0.40	0.15	0.05	-0.10
0.25	0.25	0.40	0.10
–	–	0.15	0.30
–	–	0.10	0.80

(a) Compute the expected return, the variance, and the standard deviation for each stock.

(b) On the basis of the expected return alone, which stock is preferable?

(c) On the basis of the standard deviation alone, which stock is preferable? Under what conditions can the standard deviation be used to measure the relative risk of two investments?

(d) Compute the coefficient of variation for both investments. Which stock return series has the greater dispersion? Under what conditions must the coefficient of variation be used to measure the relative risk of two investments?

Synopsis

- **An investor should examine both risk and return of an investment. Investors should not expect high return from an investment, without being prepared to undertake high risk as well.**
- **The term "return" can be used in various ways. For this reason it is important to distinguish between realized return, expected return and required return.**
- **The return of an investment consists of two parts: yield and capital gain or loss.**
- **The total return from an investment, including all sources of income, for a given period of time is called holding period return. A value of 1.0 indicates no gain or loss. The total return from an investment for a given period of time stated as a percentage is called holding period yield. Many times, an investor wants to calculate the average return of an investment which he/she has held for several years. Alternatively, an investor may wish to calculate the mean return of a portfolio of investments for a single year or for a number of years. In these cases, the investor may use one of the following statistical measures: the arithmetic mean or the geometric mean. The arithmetic mean should be used when we want to compute the average investment return for a single period or year. On the contrary, the geometric mean should be used in order to compute the average return on an investment over multiple periods.**
- **The risk of an investment is defined as the variability of the possible outcomes around their expected value or their mean. An absolute measure of risk is the standard deviation of returns (and its variance). A relative measurement of risk is the coefficient of variation.**
- **The total risk of an investment derives from various sources, the major of which are the following: interest rate risk, inflation risk, market risk, business risk, financial risk, political risk, exchange rate risk and liquidity risk. The modern investment analysis separates risk in two categories: the systematic or market risk and the unsystematic risk. The systematic risk is the investment risk which is related to movements of the entire market and cannot be eliminated by portfolio diversification. All securities, either shares or bonds, have systematic risk, as this risk includes the interest-rate risk, the market risk and the inflation risk. On the other hand, the unsystematic risk is the risk which is unique to an asset derived from its particular characteristics, and consequently, can be eliminated with portfolio diversification. The unsystematic risk includes the business risk, the financial risk and the liquidity risk.**

Answers to Activities

Activity 1

Answer:

HPR = Ending value of investment / Beginning value of investment =

(26.5 + 0.5)/25 = 1.08

HPY = HPR – 1.0 = 1.08 – 1.00 = 0.08 or 8%

Activity 2

Answer:

(a) $AM_A = [(0.0)+(0.10)+(-0.14)+(-0.05)+(0.17)]/5 = 0.28/5 = 0.0560$ or 5.6%

$AM_B = [(0.09)+(0.05)+(-0.11)+(0.04)+(0.06)]/5 = 0.13/5 = 0.0260$ or 2.6%

Stock A is more desirable because the arithmetic mean annual rate of return is higher.

(b) $\sigma_A = \{[(0.200\text{-}0.056)^2 + (0.100\text{-}0.056)^2 + (-0.140\text{-}0.056)^2 + (-0.050\text{-}0.056)^2 +$

$+ (0.170\text{-}0.056)^2]/4\}^{1/2} = (0.0853/4)^{1/2} = (0.0213)^{1/2} = 0.1460$ or 14.60%

$\sigma_B = \{[(0.090\text{-}0.026)^2 + (0.050\text{-}0.026)^2 + (-0.110\text{-}0.026)^2 + (0.040\text{-}0.026)^2 +$

$+ (0.060\text{-}0.026)^2]/4\}^{1/2} = (0.0245/4)^{1/2} = (0.0061)^{1/2} = 0.0783$ or 7.83%

By this measure stock B is preferable, because it has lower absolute risk (i.e. lower standard deviation).

(c) $CV_A = (0.1460/0.056) = 2.61$

$CV_B = (0.0783/0.026) = 3.01$

By this measure stock A is preferable, because it has lower relative risk (i.e. lower coefficient of variation).

(d) $GM_A = [(1.20)\times(1.10)\times(0.86)\times(0.95)\times(1.17)]^{1/5} - 1.0000 = (1.2618)^{1/5} - 1.000 =$

$= 1.0476 - 1.0000 = 0.0476$ or 4.76%

$GM_B = [(1.09)\times(1.05)\times(0.89)\times(1.04)\times(1.06)]^{1/5} - 1.0000 = (1.1229)^{1/5} - 1.0000 =$

$= 1.0235 - 1.0000 = 0.0235$ or 2.35%

Stock A has greater annual changes in its return than stock B (i.e. stock A has more variability than stock B). This is why the standard deviation of annual return of stock A is higher than the standard deviation of B. It is known that the greater the variability of returns, the greater the difference between the geometric and the arithmetic mean returns. Thus, the difference between the two means regarding stock A [AM_A - GM_A = 0.84] is larger than the difference in these two means of stock B [AM_B - GM_B = 0.25].

Activity 3

Answer:

(a) If the investor buys the stock A:

(P)	(HPY or r)	(P)×(r)	HPY-E(r)	$[HPY-E(r)]^2$	$(P)\times[HPY-E(r)]^2$
0.20	-0.15	-0.0300	-0.2425	0.05881	0.01176
0.15	0.00	0.0000	-0.0925	0.00856	0.00128
0.40	0.15	0.0600	0.0575	0.00331	0.00132
0.25	0.25	0.0625	0.1575	0.02481	0.00620
		E(r) = 0.0925			Sum = 0.02056

Expected return: E(r) = 0.0925 or 9.25%

Variance: $\sigma^2 = \sum(P)\times[HPY - E(r)]^2 = 0.02056$

Standard deviation: $\sigma = \{\sum(P)\times[HPY - E(r)]^2\}^{1/2} = (0.02056)^{1/2} = 0.1434$ or 14.34 %

Therefore, the expected return from the investment in stock A is 9.25% and the risk is 14.34%.

If the investor buys the stock B:

(P)	(HPY or r)	(P) × (r)	HPY - E(r)	$[HPY-E(r)]^2$	$(P)\times[HPY-E(r)]^2$
0.15	-0.50	-0.0750	-0.5400	0.29160	0.04374
0.15	-0.30	-0.0450	-0.3400	0.11560	0.01734
0.05	-0.10	-0.0050	-0.1400	0.01960	0.00098
0.40	0.10	0.0400	0.0600	0.00360	0.00144
0.15	0.30	0.0450	0.2600	0.06760	0.01014
0.10	0.80	0.0800	0.7600	0.57760	0.05776
		E(r) = 0.0400			Sum = 0.13140

Expected return: E(r) = 0.0400 or 4.00%

Variance: $\sigma^2 = \sum(P)\times[HPY - E(r)]^2 = 0.13140$

Standard deviation: $\sigma = \{\sum(P)\times[HPY - E(r)]^2\}^{1/2} = (0.13140)^{1/2} = 0.3625$ or 36.25 %

Therefore, the investor expects that his/her return from the investment in stock B will be 4.00% and the risk will be 36.25%.

(b) Stock A is preferable because of the higher expected return available [$E(r_A)$ = 9.25%] while [$E(r_B)$ = 4.00%].

(c) In general, investors are risk averse. Moreover, the lower the standard deviation of an investment, the lower its risk. Because the standard deviation of stock A [σ_A = 14.34%] is lower than that of stock B [σ_B = 36.25%], the investor will select to invest in stock A. Consequently, stock A is preferable because of the likelihood of obtaining the expected return. When two investments have the same expected return then the standard deviation can be used to measure the relative risk of these two investments. If the investments have large differences in their expected return, then the standard deviation may provide false conclusions.

(d) The coefficient of variation of the returns for both stocks are:

$$CV = \sigma/E(r)$$

$$CV_A = 0.1434/0.0925 = 1.55$$

$$CV_B = 0.3625/0.0400 = 9.06$$

The coefficient of variation of stock A is lower than that of stock B. Therefore, the coefficient of variation implies that the returns of stock B have larger relative dispersion and consequently higher risk per unit of expected return, than stock A have. The coefficient of variation can be used to measure the relative risk of two investments, when these investments have large differences in their expected return.

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CHAPTER 3

THE MONEY MARKET AND THE CAPITAL MARKET

The Scope of the Chapter

In this chapter, we help you understand the basic concepts about the money market and the capital market, as well as the characteristics of the most important securities that are traded in these markets.

Learning Objectives

When you have finished studying this chapter, you will be able to:

- compare a treasury bill with a treasury bond
- distinguish between government bonds and corporate bonds
- distinguish between a bond's coupon rate from its yield to maturity
- compare bonds that carry coupons with zero-coupon bonds.

Key Words

- Primary and secondary market
- Treasury bill
- Negotiable certificates of deposit – CDs
- Repurchase agreement
- Eurodollars
- Common stock
- Preferred stock
- Yield to maturity
- Fixed-rate bonds or floating rate bonds
- Zero-coupon bonds
- Callable bonds
- Junk bonds

Introductory Comments

In the previous chapter we discussed the concepts of risk and return. These concepts are essential for the analysis of variable-income and fixed-income securities. In this chapter we will briefly describe the main characteristics of various fixed-income and variable-income securities. In later chapters we will proceed with the evaluation and analysis of fixed-income and variable-income securities.

This chapter contains three major sections. In the first section we examine the most important instruments of the money market. In the second, we analyze the main characteristics of stocks and bonds and finally, in the third section we discuss the most important types of bonds.

3.1 THE MONEY MARKET

The money market is the market for short-term debt securities with maturity of less than one year. The money market instruments are short-term, marketable, liquid, and low default risk debt securities. Many of these securities trade in large denominations, and so are out of the reach of small investors. Money market mutual funds, however, are easily accessible to individual investors through indirect investment. These funds pool the resources of many investors and purchase a wide variety of money market securities on their behalf. The most important money market instruments are the following:

Treasury bills (T-bills). They are negotiable government securities with a maturity of less than one year that pay no periodic interest but yield the difference between their par value and their discounted purchase price. They are the most marketable of all money market instruments. They are backed by the full faith and credit of the government, as all government securities, and consequently market participants perceive Treasury bills to carry no risk of default. Investors buy the Treasury bill at a discount from the stated maturity value. At the Treasury bill's maturity, the holder receives from the government a payment equal to the face value of the bill. The difference between the purchase price and ultimate maturity value constitutes the investor's earnings. Investors can purchase Treasury bills directly at auction or on the secondary market from a government securities dealer. Treasury bills are highly liquid securities. They are easily converted to cash and sold at low transaction cost and with not much price risk.

The treasury bills have maturities of 3, 6 and 12 months (or alternatively, 13, 26 and 52 weeks). They are sold using the procedure of competitive auction, or multiple-price auction, with the participation of primary dealers. A 95% of the treasury bills that are traded in the Greek money market have maturity of 12 months. In Greece most of the Treasury bills are held to maturity. Thus, the transactions of treasury bills in the primary market are very high, while in the secondary market are quite low. Tax is computed on the difference between the face value and the issue price and is paid the day the Treasury bills are issued.

In the USA, treasury bills are sold by the use of a multiple-price auction. They have face value that lies between $1,000 and $1,000,000 and their duration is 91, 182 and 365 days. Although Treasury bills are sold at discount, their dollar yield (that is, the difference between the purchase price and the face value if the bill is held to maturity) is treated as interest income for tax purposes.

Example 1

An investor buys a treasury bill with face value €1,000 for €980. The maturity of the bill is three months. The tax rate on T-bills is 10% and tax is paid the day of issue.

(a) Which is the investor's annual return?

(b) How much of the bill's purchase price go to tax and how much towards to the present value of the bill?

Answer: (a) The investor's annual return is:

$$K_p = K_t / (1+rd) \Rightarrow 980 = 1{,}000/[1+r(90/360)] \Rightarrow r = 0.0816 \text{ or } r = 8.16\%$$

where, K_p is the money that the investor paid, K_t is the title's face value, r is the annual rate of return (or the discount interest rate) and d is the time period (expressed in years) remaining to maturity.

(b) The money that the investor paid (K_p) contains also tax (T). The tax equals the tax rate times the difference between the bill's face value (K_t) and the bill's present value (K_0); that is: $T = [t\times(K_t - K_0)]$. Therefore, the money that corresponds to the present value of the Treasury bill is:

$$K_p = K_0 + T \Rightarrow K_p = K_0 + [t\times(K_t - K_0)] \Rightarrow K_p = [K_0\times(1-t)] + (t\times K_t) \Rightarrow$$

$$K_0 = [K_p - (t\times K_t)] / (1-t)$$

Thus, $K_0 = [980 - (0.10\times 1{,}000)] / (1-0.10) \approx 977.78$ and the tax is (€980–€977.78=) €2.22.

Repurchase agreements (or repos). It is the sale of a security with a commitment by the seller to buy the security back from the purchaser at a specified price at a designated future date. Basically, the repurchase agreement is a collateralized loan, where the collateral is a security. Such loans involve very little risk to the lender, because the money market instruments typically used as in repos are of high quality. The difference between the purchase (repurchase) price and the sale price is the euro interest cost of the loan. A mirror image of the repurchase agreement is the reverse repurchase agreement. The reverse repurchase agreement (or reverse repos) is the purchase of a security with a commitment by the purchaser to sell the security back to the seller at a specified price at a designated future date. Formally, if an investor seeks for capital, the transaction is a repo, while if he/she seeks for securities, the transaction is a reverse repo.

Negotiable certificates of deposits (CDs). They are financial assets issued by the majority of banks that indicate a specified some of money has been deposited at the issuing depository institution. The CD bears a maturity date and a specified interest rate, and can be issued in any denomination. A CD may be non-negotiable or negotiable. Non negotiable CD means that the initial depositor must wait until the maturity date in order to withdraw his/her funds. An early withdrawal penalty is imposed in case the holder of CD withdraws the funds prior to the maturity date. Negotiable CD means that it can be sold by one investor to another.

Commercial papers. They are short-term unsecured promissory notes that are issued by large established corporations. In the USA, they are usually sold on a discount basis, in denominations of €100,000 or more, and their maturity is typically less than 270 days (the most common maturity range is 30 to 50 days). Typically, the investors hold the commercial papers until maturity, resulting in a very small secondary market.

Banker's acceptance. It is a security representing a bank's promise to repay a loan created in a commercial transaction in case the debtor fails to perform. Acceptances sell at a discount from the face value and they are commonly used in commercial trade transactions. In the USA they are traded in the secondary market, and their maturity lies between 30 and 180 days.

Eurodollars. They are dollar-denominated deposits held in banks or bank branches, which are located outside the United States. This market was created initially in Europe. Dollar denominated deposits can be now made in many countries outside Europe. The deposits in Eurodollars consist mainly of time deposits and certificates of deposits. Eurodollar CDs constitute the largest component of the Eurodollar market. Eurodollar securities range from overnight to five years in maturity, but most are six months or shorter. Because foreign banks are often subject to less regulations than US banks, securities issued by foreign banks usually carry larger interest payments than similar securities issued by US banks.

Federal funds. They are funds, traded in an interbank market, where only financial institutions participate. All financial institutions are required to maintain reserves deposited at the Central Bank, which are called federal funds (or fed funds). No interest is earned on federal funds. At any time some banks have more funds than required at the Fed, while other banks tend to have less than the amount required. In the federal funds market, banks with excess funds lend to those with a shortage. These loans, which are usually overnight transactions, are arranged at a rate of interest called the **federal funds rate**. The **discount rate** is the rate charged to depository institutions when they borrow reserves from the Fed to meet their reserve requirements.

3.2 THE CAPITAL MARKET

The capital market is the market for long-term financial assets with maturity of more than one year. Securities traded in the capital market usually include higher risk of default than those securities traded in the money market. This is mainly because of the longer maturity that these securities have. It is generally acceptable that, the later the payments the more likely the default probability. The most important instruments of capital markets are the stocks (common and preferred) and the bonds. These instruments can be classified by the type of financial claim into variable-income securities and into fixed-income securities. The claim that the holder of a financial asset has may be either a fixed euro amount or a varying, or a residual, amount. In the former case the financial asset is referred to as a debt instrument, or a fixed-income instrument. The financial market in which such instruments are traded is referred to as the debt market. In the latter case the financial asset is referred to as an equity instrument, or a variable-income instrument. An equity claim or a residual claim obligates the issuer of the financial asset to pay the holder an amount based on earnings, if any, after holders of debt instruments have been paid. The financial market in which such instruments are traded is referred to as the equity market, or the stock market.

3.2.1 Variable-income securities

Variable-income securities are those securities that do not pay each period (for example, semi-annually or annually) a specific amount of money to their holders, but their cash flows vary from period to period. The most important variable-income securities are the common stocks.

Common stock (or equity securities or equities) represent ownership shares in a corporation. Each share of common stock entitles its owner to a share in the financial benefits of ownership, such as the right of participating in the earnings, in the issue of new shares, and in what remains in case of liquidation. Because a common shareholder is one of the owners of the corporation, he/she is entitled to vote on matters brought up at the corporation's annual meting and to vote for the corporation's board of directors. The board, which meets only a few times each year, selects managers who actually run the corporation on a day-to-day basis.

The two most important characteristics of common stock as an investment are its residual claim and limited liability features. The former means that stockholders are the last in line of all those who have a claim on the assets and income of the corporation. The latter means that the most stockholders can lose, in the event of failure of the corporation, is their original investment.

Total return to an investor comes from dividends and capital gains, or appreciation in the stock price. These two components of return bear risk, as the shareholder's return at the end of a period may differ from the return that the shareholder expected at the beginning of that period.

Normally, firms make payments in cash to their stockholders. These payments are termed **dividends**. Occasionally, corporations pay dividends by issuing additional shares of stock. This type of distribution is called a **stock dividend**.

Dividends per share is a measure that translates total common dividends paid by the company into a per share figure. **Dividends per share** is computed as annual dividends paid to common stockholders divided by the number of common shares outstanding. A convenient way of assessing the amount of dividends received is to measure the stock's dividend yield. **Dividend yield** is the current annualized dividend paid on a share of common stock, expressed as a percentage of the current market price of the corporation's common stock. To put dividend yield into perspective, it is often helpful to look at a company's dividend payout ratio. The **dividend payout ratio** is the portion of earnings per share that a firm pays out as dividends.

Preferred stock is a class of stock, not a debt instrument, but it shares characteristics of both common stock and debt. The holder of preferred stock is entitled to dividends, which are a specified percentage of par value or face value. The percentage is called the dividend rate; it need not be fixed, but may float over the life of the issue. Preferred stock differs from bonds because its payment is a dividend and therefore not legally binding. It does not have an expiration date and, consequently, its issuer is compelled to pay the agreed dividend indefinitely. Therefore, the preferred stock resembles to the perpetual bond[1]. However, preferred dividends are usually cumulative; that is, unpaid dividends cumulate and must be paid in full before any dividends may be paid to common stockholders. The majority of preferred stock issued today, internationally, provides a call provision and is known as callable preferred stock[2]. Moreover, almost half of the preferred stock is convertible into common stock and is called convertible preferred stock[3]. Holders of a firm's preferred stock have priority over holders of common stock with respect to dividend payments and distribution of assets in the case of bankruptcy. This is why preferred stock is called a senior security. But preferred stockholders are in a more risky (or junior) position relative to the corporate bondholders. Preferred stockholders generally receive a greater rate of return on their investment than bondholders in compensation for the greater risk they bear. However, they generally receive a lesser rate of return than the common stockholders because they assume less risk. On the balance sheet, preferred stock is classified as equity.

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[1] *Perpetual bond is a bond issued without a maturity date; it pays fixed cash flows forever.*
[2] *Callable preferred stock is called the preferred stock that enables the issuer to redeem it.*
[3] *Convertible preferred stock is called the preferred stock that may, at the option of the holder, be converted into another security (usually common stock) on stated terms.*

3.2.2 Fixed-income securities

Fixed-income securities have a contractually mandated payment schedule. This means that they are securities that promise the holder to receive certain specified cash flows at predetermined times in the future. The most important fixed-income securities are the preferred stocks and the bonds. However, we should point out that bonds can also be variable-income securities, such as the floating (or variable) rate notes (FRNs). The FRN is a debt instrument that is similar to a fixed-income bond in that it pays coupons at regular (e.g. semiannual) dates during its life. The difference is that the FRN pays a coupon that is adjusted in a predetermined way with changes in some reference rate (e.g. the LIBOR).

When a firm, an organization, or even a government needs to borrow funds, they can issue fixed-income securities. Accordingly, the issuers of fixed-income securities are borrowers, while the investors who acquire fixed-income securities (except preferred stock) are really lenders to the issuers. The USA government borrows funds largely by selling Treasury notes and Treasury bonds. Treasury notes are issued with maturities from one to ten years, while Treasury bonds have maturities greater than ten years at the time of issuance. Both are issued in denominations of $1,000 or more. Both make semiannual interest payments called coupon payments, a name derived from pre-computed days, when investors would literally clip coupons attached to the bond and present a coupon to an agent of the issuing firm to receive the interest payment.

A corporate bond is a marketable legal contract that promises to pay its investor a specified percentage of par value on designated dates (known as the coupon payments) and to repay the par or the principal value of the bond at the maturity date. Bonds are the senior securities of a firm; the law requires bankrupt firms to pay off their bondholders before their stockholders[4].

The coupon, maturity, and principal value are important features of a bond. A bond's **coupon** is the periodic interest payment made to the bondholders during the life of the issue. This is known as interest income, coupon income, or nominal yield. A bond's coupon cited, is in fact the **coupon rate** (or rate of interest, or nominal yield), which, when multiplied by the par value, indicates the euro value of the coupon payment. The **term to maturity** is the date on which the debt will cease to exist and the borrower will have completely paid off the amount borrowed. In practice the words maturity and term are used interchangeably to refer to the number of years remaining in the life of a bond. The **principal**, (or the par value, or the face value) represents the amount that the issuer agrees to pay at the maturity date. In the USA, the face values of bonds are usually issued in multiples of 1,000 and range from $1,000 dollars to $25,000. However, note that the bond's face value differs from its market price. The **market price** of a bond is its value in the capital

[4] *These bonds are often called straight or plain vanilla or bullet bonds and are the most usual type of bonds.*

market, and represents the price in which the bond can be bought or sold. The market prices of many bonds rise above or fall below their face values because of the differences between their coupons and the prevailing market rate of interest. When the market interest rate is above the coupon rate, the bond sells at a discount to par. When the prevailing market interest rate is below the coupon rate, the bond sells at a premium above par. When the market interest rate is comparable to the coupon rate, the market price of the bond is close to its face value.

The coupon interest rate may differ from the return of the bondholder. The **yield to maturity (or redemption yield)** is the compounded rate of return promised to an investor who buys the bond at prevailing prices and it held until maturity. The yield to maturity is computed as the interest rate that makes the present value of a bond's payments equal to its price. The yield to maturity is a promised return, because it will only be achieved if the investor holds the bond to maturity, and reinvests all the interim cash flows at the computed yield to maturity rate. If the investor reinvests the interim cash flows at a different interest rate from the yield to maturity or spends his/her coupon's income, then the realized yield of the bond differs from the yield to maturity, even if the investor holds the bond to maturity. The **current yield** of a bond is the annual coupon payment of the bond divided by its current market price. The current yield is only one measure of return for a bond, and it is not the best because it captures only the income component of total return (i.e. the coupon interest), ignoring the capital gain or loss component. Nevertheless, the current yield is a better measure of return than that of the coupon interest rate, because it uses the current market price of the bond and not its face value.

Example 2

ABC issued in 1998 a fifteen-year bond with coupon interest rate 4% and €1,000 face value. Today this bond is sold at €800. Which is the bond's current yield?

Answer:

The annual coupon of the bond is (0.04×1,000=) €40, while the price of the bond in the market is €800. Therefore, the current yield is:

$$CY = (0.04 \times 1{,}000)/800 = 0.05 \text{ or } 5\%.$$

When a bond is issued, it may be sold in one of the following three values:

- At par value, if the coupon rate is equal to the yield to maturity. In this case, the bonds are sold at their face value. Most new-issued bonds are sold at par.
- At a premium above par value, if the coupon rate is higher than the yield to maturity. These bonds are high-coupon bonds and their market prices are higher than their face values.

- At a discount below par value, if the coupon rate is lower than the yield to maturity. These bonds are low-coupon bonds and their market prices are lower than their face values.

A bond may have a face value greater or less than €1,000. Consequently, when quoting bond prices, traders quote the price as a percentage of par value. A bond selling at par is quoted as 100, meaning 100% of its par value. A bond selling at a discount (premium) will be selling for less (more) than 100. The procedure for converting a price quote to a euro price is as follows:

$$(\text{Price per €100 of par value} / 100) \times \text{par value}$$

For example, if a bond is quoted at 98½ and has a par value of €1,000, then the euro price is

$$(98.5/100) \times €1{,}000 = €985$$

3.3 TYPES OF BONDS

The most common bonds are the following:

Fixed-rate government bonds. These bonds are issued by the government and make interest payments that are fixed until maturity. The bonds are sold using the procedure of competitive auction, or multiple-price auction, with the participation of primary dealers. Sometimes the government issues fixed-rate bonds with reopening. The bonds are paid off at their par value, the interest is received at the end of each year and the coupons of the bonds can be stripped and traded separately. All bonds are marketable securities; that is, they can be traded in the bond market. Government securities are of such high quality that their yield is often used as an example of a riskless, or default free, interest rate.

Floating–rate government bonds. They are similar to the fixed-rate government bonds, but they make interest payments that are tied to some measure of current market rates. For example, the rate might be adjusted annually to the current Treasury bill rate plus 0.5%. If the one-year Treasury bill rate at the adjustment date is 3%, the bond's coupon rate over the next year would be 3.5%.

Currency-linked government bonds. They are similar to plain vanilla bonds, but their nominal value and the interests are denominated in foreign currency. The payment of coupons and capital are made in euros based on the Euro/foreign currency exchange rate at the date of payment. Consequently, the total return of these bonds also includes the changes in the euro/foreign currency exchange rate. These bonds are particularly popular in countries where currency is relatively week.

Index-linked government bonds. They are medium-term bonds with term to maturity between 5 and 10 years, which provide a fixed annual rate of return above the inflation rate (for example 2% above the inflation rate).

Savings certificates. These bonds are bearer bonds[5] issued by the Greek government in intangible form and supplied mainly to small investors. Their maturity is at least 2 years. They are non-marketable, non-transferable and non-negotiable and cannot be used as collateral. Investors receive interest on these bonds in a lump sum at redemption. Additionally, a guaranteed minimum rate of return is offered if the bonds are held for at least five years.

Zero-coupon bonds. These bonds promise no interest payments during the life of the bond but only the payment of the principal at maturity. Therefore, the

[5] *Bearer bonds are those traded without any record of ownership. Register bonds are those that the issuer keeps records of the owner of the bonds.*

purchase price of the bond is the present value of the principal payment at the maturity date using the required discount rate for this bond. The return on the bond is the difference between what the investor pays for the bond at the time of purchase and the principal payment at maturity. For example, a bond issued in 1997 at a price of €226 will be paid off in 2007 at its face value of €1.000[6]. They are popular for three reasons. First, investors can purchase maturities that fit their particular requirements, such as a child's college fee 20 years from the time of purchase. Second, the bondholder knows from the beginning the exact rate of return he/she will earn, assuming that he/she holds it to maturity. This is because this bond does not pay the bondholder any coupons; thus, there is no coupon reinvestment risk something that the holders of other types of bonds have to face[7]. Third, when interest rates vary, the zero-coupon bonds' prices are more volatile compared to the corresponding long-term bonds with coupons. Thus, the zero-coupon bondholders can achieve high return if interest rates fall. In Greece, tax is computed in the difference between the face value and the issue price, and is paid at maturity.

Example 3

The Greek government announces the issue of a zero coupon bond with term to maturity two years and interest rate 10%. How much will an investor pay for purchasing a bond with €1,000 face value and how much will he/she receive in two years, after paying the tax, if the tax rate is 10%? Which is the after tax annual return?

Answer:
Face value: €1,000
Average fixed annual rate: 10%
Purchase price of the zero coupon bond: €826.446

[To find the purchase price of the zero coupon bond, we use the formula of the present value for two interest-bearing periods. The interests of the first year are being capitalized. Therefore, we have $1{,}000/(1+0.10)^2 = 826.446$]

Interest: (1,000-826.446 =) €173.554
Value of the bond at maturity: €1,000
Tax paid at maturity: (173.554×0.10 =) 17.355
Final amount paid to the investor at maturity: €982.665
After tax annual return: [$826.446 = 982.665/(1+r)^2 \Rightarrow r = 9.0424\% \approx$] 9.04%.

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[6] *In this case the yield to maturity of the bond is approximately 16%: [$(226)\times(1+r)^{10} = 1{,}000 \Rightarrow r \approx$ 16%].*
[7] *The bond's return also depends (among other factors) on the reinvestment rate of coupons.*

Corporate bonds. Corporate bonds bear more risk than government bonds. Thus, these bonds provide higher return to compensate for their higher risk. In the USA, corporate or municipal bonds are rated by one or more of the **bond-rating agencies**. These agencies analyze the issuing corporation and the specific issue to determine the probability of default and inform the market of their analysis via their ratings. Thus, bond ratings indicate the likelihood that a bond issuer will fall into bankruptcy. The exceptions are very few and concern either small issues, or issues from certain industries such as bank issues. These bonds are known as **non-rated bonds**. The most popular bond – rating agencies in the USA are the followings: Standard & Poor's, Moody's, Duff and Phelps, and Fitch Investors Service. These agencies assign letter grades to the bonds to reflect their assessment of the safety of the bond issue. Therefore, these agencies classify the bonds according to their default risk and actually carry out credit analysis for investors. Table 3.1 presents the various ratings assigned by two major organizations.

Table 1

Classification of bond quality ratings

Category	Standard & Poor's	Moody's
High grade bonds	AAA AA	Aaa Aa
Medium grade bonds	A BBB	A Baa
Speculative bonds	BB B	Ba B
Default bonds	CCC CC C D	Caa Ca

Convertible bonds. They have the interest and principal characteristics of other bonds, with the added feature that the bondholder has the option to turn the bond back to the firm in exchange for a specified number of common shares of the firm. This type of bond is issued in order to attract more investors than the common bond does. Sometimes, convertible bonds are callable, i.e. they can be paid off before maturity.

Callable Bonds. These are bonds where the bondholder has given the issuer the right to call the issue prior to the maturity date. An issuer will call a callable bond when the yield on bonds in the market is lower than the issue's coupon rate. In this case the bond issuer will call the issue and refund it with a lower coupon rate issue. Because of this disadvantage, callable bonds provide higher yields and often feature a period of call protection, an initial period when bonds may not be called.

Junk bonds. These are bonds with a quality rating below investment grade[8]. They are also known as high yield bonds, or speculative – grade bonds, because they are very risky and offer high possible returns to their holders. These bonds are speculative bonds and are mainly used in mergers and acquisitions of corporations.

[8] *Junk Bonds are rated at BB from Standard and Poor's and Ba from Moody's Investors Service Inc. or even lower.*

Synopsis

- **Short-term financial claims with term to maturity up to one year are traded in the money markets. Treasury bills, repurchase agreements, negotiable certificates of deposits, bankers' acceptances, commercial papers, Eurodollars and federal funds are money market instruments.**
- **Long-term financial claims with term to maturity longer than one year are traded in the capital market. The most important instruments of capital markets are the stocks (common and preferred) and the bonds. These instruments can be classified by the type of financial claim into variable-income securities and into fixed-income securities.**
- **Variable-income securities are those securities that do not pay each period (for example, semi-annually or annually) a specific amount of money to their holders, but their cash flows vary from period to period. The most important variable-income securities are the common stocks.**
- **Fixed-income securities have a contractually mandated payment schedule. This means that they are securities that promise the holder to receive certain specified cash flows at predetermined times in the future. The most important fixed-income securities are the preferred stocks and the bonds.**
- **A corporate bond is a marketable legal contract that promises to pay its investor a specified percentage of par value on designated dates (known as the coupon payments) and to repay the par or the principal value of the bond at the maturity date.**
- **The coupon, maturity, and principal value are important features of a bond.**
- **The current yield of a bond is the annual coupon payment of the bond divided by its current market price.**
- **The yield to maturity (or redemption yield) is the compounded rate of return promised to an investor who buys the bond at prevailing prices and it held until maturity. The yield to maturity is computed as the interest rate that makes the present value of a bond's payments equal to its price. The yield to maturity is a promised return, because it will only be achieved if the investor holds the bond to maturity, and reinvests all the interim cash flows at the computed yield to maturity rate.**
- **The most important types of bonds are the fixed-rate or floating-rate government bonds, the currency-linked government bonds, the index-linked government bonds, the savings certificates, the zero coupon bonds, the corporate bonds, the convertible bonds, the callable bonds and the junk bonds.**

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CHAPTER 4

FIXED-INCOME SECURITIES VALUATION

The Scope of the Chapter

In this chapter, we help you understand how you determine the value of a fixed –income security based on the present value formula, and how changes in interest rates affect the price of a fixed – income security.

Learning Objectives

When you have finished studying this chapter, you will be able to:
- calculate the intrinsic (fair) value of a bond
- approximately estimate a bond's yield to maturity
- compute the compound realized yield of a bond
- compute the duration of a bond
- estimate the bond's price variation in the market, when interest rates change.

Key Words

- Bond valuation
- Fair value
- Perpetual bond
- Yield to maturity
- Approximate yield to maturity
- Compound realized yield
- Duration
- Modified duration
- Convexity

Introductory Comments

In the previous chapter we covered the fundamentals of the fixed-income securities, the most important of which are bonds. These concepts are essential in understanding how fixed-income securities are valued. This chapter contains three sections. The first section describes how bonds are valued. The second section analyzes the relationship between interest rates and bonds. Finally, the third section investigates the duration of the bond and how we use bond duration in bond analysis.

4.1 FIXED-INCOME SECURITIES VALUATION

Valuation is the process of determining the market value of a security. The valuation process determines the security's intrinsic value. The security's purchaser pays an amount of money today in order to receive cash flows in the future. This amount of money, which is the security's market value, is equal to the present value of the future cash flows that the purchaser will receive. Thus, the future cash flows should be discounted at an appropriate rate, to determine their present value. The sum of these discounted cash flows is the security's intrinsic value. Thus, **the security's intrinsic value (or economic value or fair value or reasonable value) is the present value of the security's expected future cash flows**. When the bond market is in equilibrium, bond prices are equal to their intrinsic values. In case the market is not in equilibrium, market forces will balance it again. For example, if bond's A market price is higher than its intrinsic value, all market participants will wish to sell this bond. Thus, bond's A market price will decrease because of its high supply. This process will last until bond's A market price is equal to its intrinsic value. The opposite will happen if the bond's price is lower than its intrinsic value. In this case, all market participants will wish to buy bond A. The high demand for bond A will increase its market price. This will last until bond's A market price is equal to its intrinsic value. However, at this point we should mention that the bond market is almost always in equilibrium.
The bond's intrinsic value is:

$$IV = \frac{C}{(1+k)} + \frac{C}{(1+k)^{n}} + \ldots + \frac{C}{(1+k)^{n}} + \frac{FV}{(1+k)^{n}} \qquad (4.1)$$

where, IV = the bond's intrinsic value, C = the annual coupon payment, n = the number of years to maturity, FV = the face value (or maturity value or par value) of the bond, and k = the appropriate discount rate.

The intrinsic value computed indicates what an investor would be willing to pay for this bond to receive a rate of return that meets his expectations about various factors, such as the expected rate of inflation, the risk free rate and the risk of the bond. Each bond differs from others, which means that the required rate of return is different for each bond. This required rate of return is the discount rate used and reflects the time value of money during the purchase date and the maturity date, and the risk of the security. Furthermore, this interest rate is the bondholder's opportunity cost, since it shows the rate forgone by an investor in the next best alternative investment with comparable risk. Thus, the discount rate used to determine the bond's face value also depends on the market interest rates.

Example 1

Consider bond A, issued in the past, with three years to maturity. Its face value is €1,000 and its coupon rate 10%. The coupons are paid annually. Interest rates today are lower than the time when bond A was issued, so newly issued securities with identical characteristics yield 8%. Thus, bond's A appropriate discount rate is 8%. Find the bond's intrinsic value.

Answer:

$$IV = [100/(1+0.08)] + [100/(1+0.08)^2] + [100/(1+0.08)^3] + [1{,}000/(1+0.08)^3] \Rightarrow$$

$$IV = 1{,}051.54$$

Thus, the bond's A intrinsic value is €1,051 and this is the market price of bond A.

Several bonds pay interest semi-annually, rather than annually. In this case, the bond's intrinsic value can be found with the use of semiannual compounding as:

$$IV = \frac{C/2}{(1+k/2)^1} + \frac{C/2}{(1+k/2)^2} + \ldots + \frac{C/2}{(1+k/2)^{2n}} + \frac{FV}{(1+k/2)^{2n}} \qquad (4.2)$$

Activity 1/Chapter 4

Calculate the intrinsic value of bond A from example 4.1 (with €1,000 face value, 10% coupon rate, three years to maturity and current level of interest rates of 8%), assuming that coupons are paid twice per year.

In equations 4.1 or 4.2 the bond's intrinsic value was unknown, while all other parameters were known. However, most times the bondholder usually cares about the discount rate, while all other parameters are usually known. This is because the investor has already bought, or is going to buy, the bond at its current price and knows the coupon rate, the face value, the years to maturity and how frequently the coupons are paid. In this case, we can calculate the investor's rate of return if he holds the bond to maturity. This rate of return is known as **yield to maturity**. Thus, the yield to maturity is the interest rate that will make the present value of a bond's remaining cash flows (if held to maturity) equal to the price, and is calculated as:

$$P_0 = IV = \frac{C}{(1+YTM)^1} + \frac{C}{(1+YTM)^2} + \ldots + \frac{C}{(1+YTM)^n} + \frac{FV}{(1+YTM)^n} \qquad (4.3)$$

where, P_0 = the bond's current market value (or price), and YTM = the yield to maturity.

The yield to maturity can be calculated in several ways. The simplest way is by using a financial calculator or a PC software, such as Microsoft Excel. Another way is the method of trial-and-error (iteration), where we use several interest rates until the two parts of the above equation are equal. Finally, an approximation formula exists for calculating the yield to maturity. This approximate yield to maturity (AYTM) is computed as follows:

$$\text{AYTM} = \frac{\text{Annual coupon payment + average annual capital gain or loss}}{\text{(Bond's face value + bond's current market price)/2}}$$

or

$$\text{AYTM} = \frac{C_t + \frac{P_p - P_m}{n}}{\frac{P_p + P_m}{2}} \quad (4.4)$$

where, C_t = the annual coupon payment, P_p = the bond's face value, P_m = the bond's current market price, and n = the number of years to maturity.

The approximate yield to maturity is simple to calculate and assumes that coupons are paid once per year (i.e. annual compounding). Nevertheless, the difference between the approximate yield to maturity and the real yield to maturity is relatively small.

Example 2

An 8% - coupon bond has three years to maturity. Its face value is €1,000 and it pays interest annually. The bond's current market price is €950.26 (i.e. the bond is selling at a discount below par value). Calculate the rate of return for an investor if he/she buys this bond today and holds it to maturity.

Answer:

Using the formula of the approximate yield to maturity, we find that the required rate of return is:

AYTM = 80+[(1,000-950.26)/3] / [(1,000+950.26)/2] = 0.099 or 9.9%

This rate of return approaches the real yield to maturity, which is 10% and can be easily found by using a financial calculator or a computer.

We should point out that the yield to maturity is a promised return, because it will only be realized if the investor holds the bond to maturity and reinvests the coupons at an interest rate equal to the yield to maturity. If the investor reinvests the coupons at a different rate, the bond's realized return will differ from the yield to maturity even if the investor holds the bond to maturity. Actually, coupons are usually reinvested at different rates from the yield to maturity. In this case, the **realized compound yield** is:

$$rcy = \left[\frac{\text{Total future value of the bond}}{\text{Bond's purchase price}}\right]^{1/n} - 1.0 \qquad (4.5)$$

where, rcy = the realized compound yield, and n = the number of years to maturity[1].

Example 3

Consider an 8% - coupon bond with three years to maturity. Its face value is €1,000 and its coupons are paid annually. The bond's current market price is €950.26 (i.e. the bond is selling at a discount below par value). Calculate the realized compound yield if an investor purchases this bond today, holds it to maturity, and reinvests the coupons received from the bond at an interest rate of 12%.

Answer:

The bond's total future value is found by adding its nominal face value received at the end of the third year (i.e. €1,000) to the amount that will be accumulated at the end of the third year from the reinvestment of the coupons at an interest rate of 12%. This total amount is the future value of an ordinary annuity with periodic rent of €80, compound rate of 12% and time horizon of three years. Thus, the bond's total future value is 1,000 + [80×(3.3744)] = €1,269.95. Thus, the realized compound yield is:

rcy = [(1,269.95/950.26)$^{1/3}$] – 1.0 = (1.101492–1.0) = 0.1015 or 10.15%.

The above analysis shows that this bond will have a higher realized compound yield than the yield to maturity (that was 10%, as calculated from example 4.2).

Activity 2/Chapter 4

Calculate the realized compound yield of example 3, assuming that the coupons are paid semi-annually.

Perpetual bond is a bond without maturity which pays fixed interest forever, but it never repays the principal. Perpetuities can be illustrated by some British securities issued after the Napoleonic Wars. In 1815, the British government sold a bond issue and used the proceeds to pay off many smaller issues that had been floated in prior years to pay for the war. Since the purpose of the bond issue was to

[1] *If interest is paid semi-annually, instead of n we use 2n, where n denotes the number of periods to maturity.*

consolidate past debts, the bonds were called consols. The perpetual bond's intrinsic value is found by the following equation (4.6), resulting by the transformation of (4.1)[2]. More specifically:

$$(4.1) \quad \Rightarrow IV = \frac{C}{(1+k)} + \frac{C}{(1+k)^2} + ... + \frac{C}{(1+k)^n}$$

Multiplying both sides of equation (4.1) by (1+k) and subtracting (4.1) from the result we get:

$$IV(1+k) = C + \frac{C}{(1+k)} + \frac{C}{(1+k)^2} + ... + \frac{C}{(1+k)^n} \quad \Rightarrow$$

$$IV(1+k) - IV = C - \frac{C}{(1+k)^n}$$

As $n \to \infty$, the term $[C/(1+k)^n] \to 0$. Thus,

$$IV k = C \quad \Rightarrow \quad IV = \frac{C}{k} \qquad (4.6)$$

Example 4

Calculate the intrinsic value of a perpetual bond, if you assume that its coupon rate is 8%, its face value is €1,000 and the required rate of return is 5%.

Answer:

The bond's annual coupon is (0.08×1,000 =) €80. Thus, the bond's intrinsic value is

[IV = (80/0.05) =] €1,600.

Preferred stock can be described as a perpetual security, since it has no maturity date and will pay the indicated dividend forever. Preferred stock dividends, unlike common stock dividends, are fixed when the stock is issued and do not change. These dividends are specified as an annual euro amount, or as a percentage of par value. Because preferred stock is perpetuity, equation (4.6) is applicable in its valuation. Consequently, the intrinsic value of a preferred stock can be expressed as follows:

$$IV = \frac{D}{k} \qquad (4.7)$$

where, D = the annual preferred dividend per share.

[2] *Formula (4.6) results by using the formula of the present value of a perpetual annuity.*

Example 5

Calculate the intrinsic value of a preferred stock with a par value of €120, paying dividend of 6% of its par value. The dividends are paid annually and the appropriate required rate of return is 5%.

Answer:

The preferred stock's annual dividend is (0.06 × 120 =) €7.2. Thus, the stock's intrinsic value is [IV = (7.2/0.05) =] €144.

4.2 RELATIONSHIP BETWEEN INTEREST RATES AND BOND PRICES

The price volatility of a bond is influenced by several factors. Burton Malkiel used the above bond valuation model to demonstrate that the market price of a bond depends on the following four factors: its face value, its coupon, its maturity and the market interest rates. Malkiel derived five theorems about the relationship between bond prices and yields (interest rates)[3]. These theorems are the following:

1. Bond prices move inversely to bond yields (interest rates). A rise (decline) in interest rates will cause a decline (rise) in bond prices.
2. Bond price volatility (percentage of price change) is positively related to term to maturity. The longer the maturity, the larger the changes in bond prices, when yields (interest rates) change.
3. Bond price volatility increases at a diminishing rate as term to maturity increases. For example, if the maturity of a bond doubles, this will not result in a double percentage price change, when interest rates change.
4. Bond price movements resulting from equal absolute increases or decreases in yield are not symmetrical. A decrease in yield raises bond prices by more than an increase in yield of the same amount lowers prices.
5. Bond price volatility is inversely related to coupon. Higher coupon issues show smaller percentage price fluctuations for a given change in yield.

These bond theorems will be illustrated in the examples that follow. It has already been mentioned that the bonds' required yields are directly related to the market interest rates. As yields (required interest rates) in the market change, the only variable that can change, to compensate an investor for the new required yield in the market, is the price of the bond. When the coupon rate is equal to the required yield, the price of the bond will be equal to its face value. When the required yield in the market rises above the coupon rate at a given point in time, the price of the bond adjusts so that an investor contemplating the purchase of the bond can realize some additional interest. If the bond price did not change, investors would not buy the issue because it offers a below market yield. The resulting lack of demand would cause the bond price to fall and thus the yield on the bond to increase. This is how a bond's price falls below its face value, and it is said to be selling at a discount. When the yields in the market fall below the coupon rate, the bond must sell above its face value, and is said to be selling at a premium. This is because investors who have the opportunity to purchase the bond at face

[3] *For more information see Malkiel (1962).*

value would getting a coupon rate in excess of what the market requires. In consequence, investors would bid up the price of the bond because its yield is so attractive. The bond price would eventually be bid up to a level where the bond offers the required yield in the market.

From the above it follows that if interest rates decrease, bond prices will increase, so that these (outstanding) bonds can provide similar yields (i.e. required rates of return) to their possible buyers, compared with the newly issued bonds. On the contrary, if interest rates increase, bond prices will decrease, so that these (outstanding) bonds can provide similar yields (i.e. required rates of return) to their possible buyers, compared with the newly issued bond yields.

Example 6

An investor purchases a five year 12% coupon bond at par value paying €1.000. The bond makes annual interest payments. After the second interest payment, the investor decides to sell the bond. At this time the interest rates have declined to 9%. (a) Which is the new bond market price? (b) If the investor sells the bond at the new price, which is his realized rate of return?

Answer:

(a) If the bondholder kept the bond to maturity (i.e. another three years), the yield to maturity would be 12%. If interest rates had not been changed and new issued bonds with similar characteristics yielded 12%, the investor would sell his bond for €1,000. However, new issued bonds yield 9% and consequently the given bond's market price should increase. The new price should be:

$$P_0 = IV = [120/(1+0.09)]+[120/(1+0.09)^2]+[1{,}120/(1+0.09)^3] = 1{,}075.94$$

Thus, the bond price moved inversely to its yield to maturity and consequently, this example demonstrates Malkiel's first bond theorem.

(b) Since the investor decided to sell the bond, he/she will sell it for €1,075.94. In this case, the realized rate of return is:

$$1{,}000 = [120/(1+k)]+[120/(1+k)^2]+[1{,}075.94/(1+k)^2]$$

Solving for k, we find that the realized yield is about 15.52%. This yield is higher than the yield to maturity, because the investor sold the bond at a higher price than its face value.

Activity 3/Chapter 4

An investor buys a bond at a market price that is equal to its par value, paying €1,000. This bond has a 12% coupon rate, a term to maturity of five years, and makes annual interest payments. After the second interest payment, the investor decides to sell the bond. At this time the interest rates have risen to 15%. (a) Which is the new bond market price? (b) If the investor sells the bond at the new price, which is his/her realized rate of return? (c) Compare (in absolute sizes) the bond price decrease, due to the interest rates increase from 12% to 15%, with the bond price increase from example 6, due to the interest rates decrease from 12% to 9%.

The prices of bonds with different terms to maturity react differently when interest rates change. An increase (decrease) in interest rates results in a decrease (increase) in bond prices. However, the longer maturity bond experienced the greater price volatility, assuming that all others remain the same. The holder of a bond with longer term to maturity receives the largest part of his/her cash flows over a longer time period than the holder of a bond with shorter term to maturity. Since bond prices are determined by discounting the future cash flows, a change in the discount rate causes higher bond price volatility for the bonds whose cash flows are further in the future, than those whose cash flows are closer to the present. Thus, **for a given coupon rate and initial yield, the longer the tern to maturity, the greater the bond price volatility, due to changes in the required yield.**

Example 7

Consider two bonds, one (called A) with three years to maturity and the other (called B) with twenty years to maturity. Both have €1,000 face values and 7% coupon rates (with annual interest payments), and both sell at par. An investor purchases these bonds today. A year later, the yields of both bonds rise from 7% to 9%. Calculate the new bond prices, and the percentage price changes of the two bonds.

Answer:

A year has elapsed since bonds A and B were bought. Consequently, bond A has a term to maturity of 2 years and bond B of 19 years. The current required rate of return is 9%. Bonds' A and B prices will be equal to their intrinsic value, thus, they will be equal to the present value of their total cash inflows. Consequently:

$$P_0 = IV = \frac{C}{(1+k)^1} + \frac{C}{(1+k)^2} + \dots + \frac{C}{(1+k)^n} + \frac{FV}{(1+k)^n}$$

$$P_0 = \sum_{t=1}^{2} = \frac{C}{(1+0.09)^t} + \frac{C}{(1+0.09)^2} = 964.837 \approx 964.84$$

$$P_0 = \sum_{t=1}^{19} = \frac{70}{(1+0.09)^t} + \frac{1{,}000}{(1+0.09)^{19}} = 964.837 \approx 964.84$$

From the above it follows that the price volatility of the bond with the longest term to maturity (i.e. bond B) is greater than the price volatility of the bond with the shortest maturity (i.e. bond A), due to interest rates changes. The price decrease of bond A is (1,000-964.84 =) €35.16, while that of bond B is (1,000-821.01 =) €178.99. Thus, the bond A's percentage price decrease is (35.16/1.000 =) 3.516%, while that of bond B's is (178.99/1,000 =) 17.899%. This example demonstrates the second and the third theorem of B. Malkiel. Bond B has longer term to maturity than A, and thus has also larger price volatility. However, although bond B has almost ten times longer maturity than bond A, the percentage price volatility of bond B was not ten times larger than of bond A, but almost five times larger.

Except for term to maturity, bond price volatility (owing to interest rates changes), also depends on the coupon rate. In other words, the prices of bonds which differ only according to their coupon rates react differently to interest rates changes. An increase (decrease) in interest rates results in a decrease (increase) in bonds prices. However, bond price volatility is larger, when coupons are lower. Low-coupon bonds provide the larger part of their cash inflows in the form of the principal paid to the bondholder at the maturity date, while they provide the smaller part of their cash inflows in the form of coupon payments made over the life of the bond. The opposite occurs in the case of the high-coupon bonds. Consequently, the low-coupon bondholder receives the larger part of his/her cash inflows later than the high-coupon bondholder. Since bond market prices are determined by discounting the future cash inflows, a change in the discount rate will cause larger price volatility for bonds whose higher cash inflows are received further in the future than those whose higher cash inflows are received closer to the present. Thus, **for a given term to maturity and initial yield, the lower the coupon rate the greater the price volatility of a bond.**

Example 8

An investor buys today two bonds, A and B, which are alike in all respects except coupon rate. Bond A is newly issued with 10% coupon rate, while bond B has been issued in the past with 20% coupon rate. The coupon payments are come once annually. Both bonds have a maturity of 7 years and face value of €1,000. The investor pays €1,000 to buy bond A and €1,486.88 to buy bond B. Thus, bond A is selling at par value, while bond B is selling above par value. Both bonds provide a yield to maturity of 10%. One year after the bonds' purchase, interest rates fall to 5%. Consequently, bonds' A and B market prices should increase in order to provide similar yields compared with the new-issued securities with the same characteristics. Calculate the bond prices if

interest rates had not been changed. Which bond has provided the greatest percentage price volatility due to interest rates falling from 10% to 5%?

Answer:

Both bonds have a remaining term to maturity of 6 years. The current required rate of return is 5%. Bonds' A and B prices will be equal to their intrinsic value, thus equal to the present value of their total cash inflows:

$$P_A = \sum_{t=1}^{6} = \frac{100}{(1+0.05)^t} + \frac{1{,}000}{(1+0.05)^6} = 1{,}253.77$$

$$P_B = \sum_{t=1}^{6} = \frac{200}{(1+0.05)^t} + \frac{1{,}000}{(1+0.05)^6} = 1{,}761.34$$

If interest rates remained at 10%, bond A would be selling at par value (i.e. €1,000), while bond B would be selling above par value:

$$P_B = \sum_{t=1}^{6} = \frac{200}{(1+0.10)^t} + \frac{1{,}000}{(1+0.10)^6} = 1{,}435.56$$

We note that bond A's price increased, because interest rates decreased, by (1,253.77-1,000=) €253.77 or (253.77/1,000=) 25.38%. Bond B's price increased by (1.761,34-1.435,56=) €325.78 or (325.78/1,435.56 =) 22.69%. Thus, bond A's price volatility is larger than bond B's price volatility. In general, the price of low-coupon bonds exhibits greater sensitivity to changes in interest rates, than prices of high-coupon bonds. This example is consistent with the fifth theorem of B. Malkiel.

From the above it follows that **a decline (rise) in interest rates will cause a rise (decline) in bond prices, with the most volatility in bond prices occurring in longer-maturity bonds and bonds with low coupon rates.** Consequently, price volatility depends on the combination of the coupon rate and the term to maturity. These two factors (i.e. the coupon rate and the maturity of the bond) are related to the **interest rate risk**. As we have mentioned in the previous chapter, interest rate risk is the uncertainty in the return on a fixed-income security caused by unanticipated fluctuations in the value of the asset owing to changes in interest rates. Thus, an investor, who expects a major decline in interest rates, he/she should attempt to build a portfolio of long-maturity bonds with low coupons. In this case the investor wants a bond portfolio with the maximum interest rate sensitivity in order to enjoy the maximum price changes (capital gains) from the change in interest rates. In contrast, if the investor expects a rise in interest rates, he/she would change the portfolio to short-maturity bonds with high coupons. In this case the investor wants a bond portfolio with the minimum interest rate sensitivity in order to minimize the capital losses caused by the increase in market interest rates. Because the bond price volatility varies inversely with its coupon rate and directly with maturity, it is necessary to determine the best combination of these two variables to accomplice the investor's objective. This effort would benefit from a measure that would take into account both coupon rate and term to maturity. Such a measure, called duration, is available.

Activity 4/Chapter 4

An investor purchases today two bonds, A and B, which are alike in all respects except coupon rate. Bond A has a coupon-rate of 10%, while bond B has one of 16%. Both bonds make annual interest payments, have €1,000 face value and a term to maturity of six years. The investor pays €1,000 to buy bond A and €1,261.35 to buy bond B. Thus, bond A is selling at par value, while bond B is selling above par value. Both bonds are priced in the market in such a way that both securities provide a yield to maturity of 10%. One year after the securities are purchased, the yields of both bonds fall to 8%. (a) Calculate the new bond market prices, if the yields remained unchanged. (b) Which bond has the higher price volatility caused by the decrease in interest rates from 10% to 8%? (c) Which will be the yield for the investor if he/she sells bond A and which if he/she sells bond B? Which bond would you recommend him/her to sell?

4.3 DURATION

4.3.1 Determining duration

We have already seen that two bonds which are alike in all respects except coupon rate do not have the identical economic lifetimes. The high-coupon bondholder will receive a big part of his/her return sooner than the low-coupon bondholder. The same happens when two bonds differ regarding only their maturity. The holder of short-term bonds will receive his/her return sooner than the holder of long-term bonds. Therefore we need a composite measure that will consider both coupon and maturity. Such composite measure of the interest rate sensitivity of a bond is available, and is referred to as **duration**. This concept and its development as a tool in bond analysis exist for more than 60 years. Duration is a measure of the "average maturity" of the stream of payments associated with a bond. More specifically, **duration is the weighted average term to maturity of the cash flows from the bond, where the weights are the present value of the cash flow divided by the bond market price. It is commonly referred to as Macaulay duration, after its discoverer[4].** Note that duration is measured in years.

Duration is a more appropriate measure of time characteristics than the term to maturity, because it considers both the repayment of capital at maturity and the size and timing of coupon payments prior to final maturity. On the contrary, term to maturity provides only the time interval until the last cash inflow.

The duration can be easily calculated, as follows:

- We calculate the present value of the bond's cash inflows for each year, by using as a discount rate the yield to maturity of the bond.
- We transform the above present values of each year into percentages of the total present value of the bond (i.e. its market price).
- We multiply the above percentages of each year by the number of year that each payment is to be received and we add up.

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[4] *Another definition that is often cited in the literature is that duration is the number of years needed to fully recover purchase price of bond, given present values of its cash flows. These definitions are based on the initial use of the term from Macaulay (1938), who used this measure rather than maturity as a proxy for the average length of time that a bond investment is outstanding.*

According to the above, duration is:

$$D = \sum_{t=1}^{N} t \left[\frac{\frac{c_t}{(1+k)^t}}{\sum_{t=1}^{N} \frac{c_t}{(1+k)^t}} \right] \tag{4.8}$$

where, D = the bond's duration, c_t = the cash inflows (interest or principal payment) in period t, k = the yield to maturity, and t = the time period in which the cash flow occurs.

Example 9

An investor buys a bond with three years term to maturity, €1,000 face value and 10% coupon-rate. The coupons are paid annually. The investor paid €1,051.50 to buy the bond which means that the yield to maturity is 8%. Calculate the duration of the bond.

Year (1)	Cash inflows (2)	Discount factor at 8% (3)	Present value (4) = (2) × (3)	(5) = (4)/Price	(6) = (1) × (5)
1	100	0.9259	92.59	0.0881	0.0881
2	100	0.8573	85.73	0.0815	0.1630
3	1,100	0.7938	+873.18	+0.8304	+2.4912
			Price = 1,051.50	1.0000	D = 2.7423

Thus, the bond's duration is 2.7423 years, and shows the number of years needed to recover the purchase price of the bond, given the present values of its cash flows.

The duration is important in bond analysis and bond management for the following three main reasons:

- First, duration is a measure of the bond's effective life. For example, suppose that we compare two bonds with €1,000 face value and 8% coupon-rate. The first matures in 10 years and the second in 50 years. The second bond has five times longer maturity than the first. However, the first bond has an effective life (duration) 7.25, while the second 13.21. Thus, these two bonds do not differ as much as it may seem. The reason for the sharp differences between maturity and duration is that the cash receipts received in the distant future have very small present values and therefore add little to a bond's value.
- Second, duration can be used in various bond management strategies, particularly immunization.

- Third, duration is a measure of bond price sensitivity to interest rates changes. In other words, duration measures the bonds' interest rate risk. This topic will be examined later in this chapter.

Example 10

Consider a bond selling at its par value of €1,000, with five years to maturity and an 8% coupon-rate. The coupons are paid semi - annually. Calculate the bond's duration.

Answer:

The annual coupon rate is 8%, while the semi-annual rate is 4%. Therefore, the investor receives €40 every six months. The bond's duration is:

Year (1)	Cash inflows (2)	Discount factor at 4% (3)	Present value (4)=(2)×(3)	(5)=(4)/Price	(6)=(1)×(5)
0.5	40	0.9615	38.46	0.03846	0.0192
1.0	40	0.9246	36.984	0.03698	0.0370
1.5	40	0.8890	35.56	0.03556	0.0533
2.0	40	0.8548	34.192	0.03419	0.0684
2.5	40	0.8219	32.876	0.03288	0.0822
3.0	40	0.7903	31.612	0.03161	0.0948
3.5	40	0.7599	30.396	0.03040	0.1064
4.0	40	0.7307	29.228	0.02923	0.1169
4.5	40	0.7026	28.104	0.02810	0.1265
5.0	1.040	0.6756	+702.624≈	+0.70262≈	3.5131
			1,000=Price	1.00000	D=4.2178

Thus, the bond's duration is 4.2178 years. This number shows the average maturity of the stream of payments associated with this bond. Had we used periods (i.e. 1, 2, 3, ..., 10) instead of years in the first column, we will end up with the duration shown in half years, because the cash flows of the bond occur every six months. In this case, to adjust the duration to an annual figure, the duration must be divided by 2. In general, if the cash flows occur every m times per year, the durations are adjusted by dividing by m.

Activity 5/Chapter 4

You manage the portfolio of a mutual fund and you are offered the following bonds:

Bonds	Coupon rate	Maturity (in years)	Market price (€)	Yield to maturity
A (new issue)	14%	20	1,016.80	13.75%
B (outstanding issue)	6%	20	484.80	13.60%

You expect a decline in interest rates over the next three years. Identify and justify which of these bonds you would select.

4.3.2 Characteristics of duration

There is a consistency between the properties of bond price volatility we discussed earlier and the properties of Macaulay duration. The main characteristics of Macaulay duration, which hold when all other factors are constant, are the following:

1. There is an inverse relationship between duration and coupon. Bonds with larger coupons will have a shorter duration because more of the total cash flows come earlier in the form of interest payments.
2. There is an inverse relationship between duration and yield to maturity. Higher yields result in lower present values of cash flows received later, consequently decreasing their relative value and thus the bond's duration.
3. There is generally a positive relationship between duration and term to maturity, but duration increases at a decreasing rate with maturity, particularly when maturity is longer than 15 years. Thus, the longer the maturity, the greater the duration. However, the relationship is not direct because as maturity increases, the present value of the principal declines in value. Notice that the duration of bonds selling at deep discount (i.e. bonds with 3% coupon-rate, and 14% yield to maturity), declines at very long maturities (i.e. over twenty years).
4. The duration of zero-coupon bonds is always equal to their maturity, because the only cash flow is paid at maturity.
5. The duration of bonds that pay coupons is always shorter than maturity, because duration gives weight to these interim interest payments.
6. The duration of perpetual bonds is only affected by yield to maturity and is independent of their coupon. For example, the duration of a perpetual bond which provides 6% yield to maturity, is 17.667 years, independent of its coupon rate (2%, 4%, or any other).

4.3.3 Modified duration

A bond's **interest rate risk** can be measured by its price elasticity as:

$$\text{EL} = \frac{\text{Percentage change in bond's price}}{\text{Percentage change in }(1.00+\text{YTM})}$$

where, EL = the bond's price elasticity, and YTM = the bond's yield to maturity.

For non-callable bonds, price elasticity is always a negative number, because bond prices move inversely to yield to maturity. Thus, duration and price elasticity are equivalent measures of the interest rate risk of a bond. Both measure the bond price sensitivity to changes in the market interest rates. The two formulas produce the same numerical answer. However, these two measures sometimes provide slightly different results; thus, several analysts prefer to use a modified form of duration in order to capture the bond price volatility to interest rates changes. This measure is called **modified duration**. It is an adjusted measure of duration and can be used to approximate the interest rate sensitivity of a non-callable bond. The formula for calculating the modified duration, denoted D_{mod}, is:

$$D_{\text{mod}} = Modified \quad Duration = \frac{Macaulay's \quad Duration}{\left(1+\frac{k}{m}\right)} \qquad (4.9)$$

where, k = the bond's yield to maturity, and m = the number of payments in a year. The term "m" is usually either 1 or 2, because coupons are usually paid either annually or semi-annually.

It has been shown, both theoretically and empirically, that price movements of non-callable bonds vary proportionally (but in an opposite direction) with modified duration for small changes in yields. More specifically, an estimate of the percentage change in bond price equals the change in yield times modified duration:

$$\frac{\Delta P}{P_0} \approx \frac{-D}{\left(1+\frac{k_0}{m}\right)} \times \Delta k \times 100 \qquad (4.10)$$

where, $\Delta P = (P_1 - P_0)$ = the change in price for the bond, P_0 = the beginning price for the bond, P_1 = the new price for the bond, D = the duration of Macaulay, k_0 = the yield to maturity that corresponds to the initial interest rate, k_1 = the new yield to maturity, $\Delta k = (k_1 - k_0)$ = the yield change in decimal form. For example, if yields increase instantaneously from 10% to 10.5%, then Δk = (0.105-0.100 =) 0.005.

The above equation shows that bond price volatility is proportional to duration. Consequently, the prices of two bonds with the same duration will cause the same percentage price change for small changes in required yield.

Example 11

Consider the bond from example 9 (with €1,000 face value, 10% coupon-rate, 3 years term to maturity and €1,051.50 market price). This bond has 8% yield to maturity and 2.7423 years duration. Using duration calculate the percentage price change in the bond if interest-rates decrease from 8% to 7.50% (a 50-basis-point decrease).

Answer:

$$\frac{\Delta P}{P_0} \approx \frac{-2.7423}{(1+0.08)} \times (0.075 - 0.080) \times 100 =$$

$$= (-2.5392) \times (-0.0050) \times 100 = 1.2696$$

The above equation shows that the bond's modified duration is 2.5392 years and that the bond price will increase by approximately 1.27%. If the price of the bond before the decline in interest rates was €1,051.50, the price after the decline in interest rates should be approximately [(1,051.50)×(1.012696)=1,064.8498 ≈] €1,064.85. If the price is €1,064.85, a possible bond's purchaser will enjoy a yield to maturity of 7.5% which is the yield that bonds with the same characteristic provide.

Activity 6/Chapter 4

An investor buys today a bond with three years term to maturity, €1,000 face value and 7% coupon rate (with annual interest payments). The yield to maturity is 6%. (a) Calculate the market price of the bond. (b) Calculate the bond's duration. (c) Using duration calculate the percentage price change in bond if interest rates increase from 6% to 6.20% (a 20-basis-point increase). Which will be the new bond price?

Modified duration allows us to estimate bond price changes for a change in interest rates. However, the above equation is accurate only for very small changes in interest rates. The accuracy of the estimate of the bond price change deteriorates with larger changes in interest rates. Duration will overestimate the bond price change when the required yield rises, thereby underestimating the new bond price. When the required yield falls, duration will underestimate the bond price change and thereby underestimate the new bond price. This happens because modified duration provides symmetric percentage price change. This means that the above equation is a linear approximation of a bond price change, whereas the actual relationship between bond price and yield to maturity is curvilinear. The **bond's convexity** is a measure of the curvature of a bond's price – yield relationship. Mathematically, convexity is the second derivative of price with respect to yield (d^2P/dk^2), divided by price (P). In other words, convexity is the percentage change in (dP/dk), for a given change in yield.

From the above it follows that for large changes in yields, we will consider both the bond convexity, and modified duration to estimate the bond price changes.

The preceding analysis regarding the duration drives us to the following conclusions. If an investor wishes high volatility from a bond, he/she should choose the one with the longest duration. On the contrary, if he/she wishes low volatility, he/she should choose that bond with the shortest duration. If an investor already owns a bond portfolio and expects a decline in interest rates, he/she should act to increase the duration of the portfolio, so as to achieve the largest price appreciation possible. On the contrary, if an investor who owns a bond portfolio expects a rise in interest rates, he/she should act to decrease the duration of the portfolio in order to minimize the loss from the decline in the value of the bond portfolio. Notice that duration is additive, which means that the duration of a portfolio is simply the weighted average duration of the bonds in the portfolio. The weight is the proportion of the portfolio that a security comprises. Mathematically, a bond portfolio's duration can be calculated as follows[5]:

$$D_p = W_1 D_1 + W_2 D_2 + W_3 D_3 + \ldots + W_k D_k \qquad (4.11)$$

where, D_p = the portfolio duration, W_i = the bond i market value divided by the portfolio market value, D_i = the duration of bond i, k = the number of bonds in the portfolio.

Activity 7/Chapter 4

You expect a decline in interest rates over the next six months. (a) Given your interest rate outlook, state what kinds of bonds you want in your portfolio in terms of duration and explain your reasoning for this choice. (b) You must make a choice between the following three sets of non-callable bonds. In each case, select the bond that would be best for your portfolio given your interest rate outlook and the consequent strategy set forth in (a). In each case briefly discuss why you selected the bond. [Note: First compare bonds A and B. Second compare bonds C and D. Finally compare bonds E and F.]

	Bonds	Maturity (in years)	Coupon rate	Yield to maturity
Case 1	A	15	10%	10%
	B	15	6%	8%
Case 2	C	15	6%	10%
	D	10	8%	10%
Case 3	E	12	12%	12%
	F	15	12%	8%

[5] *For more information see Fabozzi (1996), p.283.*

Synopsis

- **Valuation is the process of determining the market value of a security.**
- **The security's intrinsic value is the present value of the security's expected future cash flows from that asset.**
- **The yield to maturity is the interest rate that will make the present value of a bond's remaining cash flows (if held to maturity) equal to the price. The yield to maturity is a promised return, because it will only be realized if the investor holds the bond to maturity and reinvests the coupons at an interest rate equal to the yield to maturity.**
- **Perpetual bond is a bond without maturity which pays fixed interest forever, but it never repays the principal.**
- **There is an inverse relationship between bond prices and interest rates (yields). A decline (rise) in interest rates will cause a rise (decline) in bond prices, with the most volatility in bond prices occurring in longer-maturity bonds and bonds with low coupon rates.**
- **Duration is a measure of the "average maturity" of the stream of payments associated with a bond. More specifically, duration is the weighted average term to maturity of the cash flows from the bond, where the weights are the present value of the cash flow divided by the bond market price. It is commonly referred to as Macaulay duration, after its discoverer. The duration is a more appropriate measure of time characteristics than the term to maturity, because it considers both the repayment of capital at maturity and the size and timing of coupon payments prior to final maturity.**
- **Duration is inversely related to coupon and yield to maturity.**
- **There is generally a positive relationship between duration and term to maturity, but duration increases at a decreasing rate with maturity, particularly when maturity is longer than 15 years**
- **Modified duration can be used to capture the bond price volatility to interest rate changes. Modified duration equals Macaulay duration divided by 1 plus the current yield to maturity divided by the number of payments in a year.**
- **Price movements of non-callable bonds vary proportionally (but in an opposite direction) with modified duration for small changes in yields. For small changes in the required yield, modified duration gives good approximation of the percentage change in bond price. As the changes in the required yield become larger the approximation becomes poorer. This happens because modified duration provides symmetric percentage price change. This means that the above equation is a linear approximation of a bond price change, whereas the actual relationship between bond price and yield to maturity is curvilinear. The bond's convexity is a measure of the curvature of a bond's price – yield relationship.**

Answers to Activities

Activity 1

Answer:

The bond matures in three years or six semi-annual periods. Each semi - annual coupon payment will be (100/2=) €50. The appropriate discount rate is 8% per year or 4% per six months. Therefore, bond's A intrinsic value is:

$$IV = [50/(1+0.04)] + [50/(1+0.04)^2] + ... + [50/(1+0.04)^6] + [1{,}000/(1+0.04)^6] \Rightarrow$$

$$IV = 1{,}052.421 \approx 1{,}052$$

Thus, bond's A intrinsic value is approximately €1,052 and this is the market price of bond A.

Activity 2

Answer:

The bond matures in three years or six semi-annual periods. Each semi – annual coupon payment will be (80/2 =) €40. The appropriate reinvestment rate is (0.12/2 =) 6% per six months. The bond's total future value is found by adding its nominal face value received at the end of the third year (i.e. €1,000) to the amount that will be accumulated at the end of the third year from the reinvestment of the coupons at an interest rate of 6% per semi-annual period. This total amount is the future value of an ordinary annuity with periodic rent of €40, compound rate of 6% and time horizon of six periods. Thus, the bond's total future value is 1,000 + [40×(6.9753)] = €1,279.01. Therefore the realized compound yield will be:

$$rcy = [\ (1{,}279.01/950.26)^{1/6}] - 1.0 = (1.050764 - 1.0) = 0.050764 \text{ or } 5.0764\%$$

on a semi-annual basis. To place this on an annual basis, multiply by 2. The annual rcy is (5.0764×2=) 10.1528%.

Activity 3

Answer:

(a) If the bondholder kept the bond to maturity (i.e. another three years), the yield to maturity would be 12%. If interest rates had not been changed and newly issued bonds with similar characteristics yielded 12%, the investor would sell his bond for €1,000. However, newly issued bonds yield 15% and consequently the given bond's market price should decrease. The new price should be:

$$P_0 = IV = [120/(1+0.15)]+[120/(1+0.15)^2]+[1{,}120/(1+0.15)^3] = €931.50$$

Thus, the bond price moved inversely to its yield to maturity and consequently, this example demonstrates B. Malkiel's first bond theorem.

(b) Since the investor decided to sell the bond, he/she will sell it for €931.50. In this case, the realized rate of return is:

$$1{,}000 = [120/(1+k)] + [120/(1+k)^2] + [931.50/(1+k)^2]$$

Solving for k, we find that the realized yield is about 8.72%. This yield is lower than the yield to maturity, because the investor sold the bond at a lower price than its face value.

(c) The bond price decrease is (1,000-931.50=) €68.50. The bond price increase from example 4.6 was (1,075.94-1,000 =) €75.94. This finding demonstrates Malkiel's fourth bond theorem. A decrease in yield (from 12% to 9%) raises bond prices by more than an increase in yield (from 12% to 15%) of the same amount lowers bond prices.

Activity 4

Answer:

(a) Both bonds have a remaining term to maturity of 5 years. The current required rate of return is 8%. Bonds' A and B prices will be equal to their intrinsic value, thus equal to the present value of their total cash inflows:

$$P_A = \sum_{t=1}^{5} \frac{100}{(1+0.08)^t} + \frac{1{,}000}{(1+0.08)^5} = 1{,}079.87$$

$$P_B = \sum_{t=1}^{5} \frac{160}{(1+0.08)^t} + \frac{1{,}000}{(1+0.08)^5} = 1{,}319.432$$

Thus, bond's A price will be €1,079.87 and bond's B price will be €1,319.432.

(b) If interest rates remained at 10%, bond A would be selling at par value (i.e. €1,000), while bond B would be selling above par value:

$$P_B = \sum_{t=1}^{5} \frac{160}{(1+0.10)^t} + \frac{1{,}000}{(1+0.10)^5} = 1{,}227.428$$

We note that bond A's price increased, because interest rates decreased, by (1,079.87-1,000=) €79.87, or (79.87/1,000=) 7.99%. Bond B's price increased by (1,319.432-1,227.428=) €92.004 or (92.004/1,227.428=) 7.50%. Thus, bond A's price volatility is larger than bond B's price volatility. In general, prices of low-coupon bonds exhibit greater sensitivity to changes in interest rates, than prices of high-coupon bonds. This example is consistent with the fifth theorem of B. Malkiel.

(c) The investor paid €1,000 to buy bond A and €1,261.348 to buy bond B. Furthermore, the investor received €100 as coupon payment from bond A and €160 from bond B. If the investor sells the bonds he/she will receive €1,079.87 from bond A and €1,319.432 from bond B. In this case, his/her yields will be:

$$r_A = \frac{C_1 + P_1 - P_0}{P_0} + \frac{100 + 1{,}079.87 - 1{,}000}{1{,}000} = 0.1799$$

$$r_B = \frac{C_1 + P_1 - P_0}{P_0} + \frac{160 + 1{,}319.432 - 1{,}261.348}{1{,}261.348} = 0{,}1729$$

Hence, if the investor sells bond A his/her yield would be 17.99%, but if he/she sells bond B, his/her yield would be 17.29%. In consequence, we would recommend the investor to sell bond A.

Activity 5

Answer:
Bond B (outstanding issue) would be preferable to bond A (new issue) for the following reasons:

(a) Bond B has **longer duration** than bond A, because the coupon rate of bond B is lower than that of bond A. This means that the price volatility of bond B is greater than that of bond A, when interest rate changes. Because interest rates are expected to decline over the next three years, bond's B price will increase more than the increase in bond's A price. Thus, bond B will cause a higher increase in the portfolio value than bond A.

(b) Bond B has **less reinvestment risk** than bond A, because it pays its bondholders lower coupons (that should be reinvested at an interest rate of 13.60%) and, consequently, more of the return comes from the price change over time, which is assumed to increase at the yield to maturity rate. On the contrary, bond's A coupons (which are higher than bond's B coupons) should be reinvested at an interest rate equal to 13.75%. However, this interest rate will be particularly difficult to be achieved as interest rates are expected to fall over the next three years.

Activity 6

Answer:

Year (1)	Cash inflows (2)	Discount factor at 6% (3)	Present values (4) = (2)×(3)	(5)=(4)/Price	(6)=(1)×(5)
1	70	0.9434	66.04	0.06432	0.06432
2	70	0.8900	62.30	0.06068	0.12136
3	1,070	0.8396	+898.37	+0.87500	+2.62500
			Price=1,026.7	1.0000	D=2.81068

(a) Thus, the bond's price is €1,026.71.

(b) The bond's duration is 2.81068 years, and shows the number of years needed to recover the purchase price of the bond, given the present values of its cash flows.

(c) $$\frac{\Delta P}{P_0} \approx \frac{-2{,}81068}{(1+0{,}06)} \times (0{,}062 - 0{,}060) \times 100 = (-2{,}6516) \times (-0{,}0020) \times 100 = -0{,}5303$$

The above equation shows that the bond's modified duration is 2.6516 years and that the bond price will decrease by approximately 0.53%. If the price of the bond before the increase in interest rates was €1,026.71, the price after the increase in interest rates should be approximately [1,026.71 × (1.0000-0.0053) = 1,021.2684 ≈] €1,021.27. If the price is €1,021.2, a possible bond's purchaser will enjoy a yield to maturity of 6.2% which is the yield that bonds with the same characteristic provide.

Activity 7

Answer:

(a) Given that you expect a decline in interest rates over the next six months, you should choose bonds with the maximum price volatility; that is, you should choose bonds with the longest duration.

(b) Case 1. Given a choice between bonds A and B you should select bond B that has the lower coupon rate and the lower yield to maturity. This is the best choice because of the inverse relationship between duration and both coupon and yield to maturity.

Case 2. Given a choice between bonds C and D you should select C. The duration is positively related to yield to maturity and inversely related to coupon.

Case 3. Given a choice between bonds E and F you should select F. The duration is positively related to maturity and inversely related to yield to maturity.

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CHAPTER 5

STOCK VALUATION

The Scope of the Chapter

In this chapter, we help you understand how the market determines the value of a stock and how you could estimate the major inputs to the stock valuation models.

Learning Objectives

When you have finished studying this chapter, you will be able to:

- estimate the intrinsic value of a stock using two different models
- describe the three factors that affect the earnings multiplier of a firm
- mention the two factors that affect the dividend growth rate
- compute the expected earnings and dividend growth rate of a stock
- define the two factors which affect the expected earnings per share
- calculate the expected return of a stock
- mention three ratios (except for the P/E ratio) which are used during the stock valuation procedure.

Key Words

- Dividend payout ratio
- Retention rate
- Dividend yield
- Stock's intrinsic value
- Earnings multiplier or P/E ratio
- Dividend discount model
- Price to book value ratio – P/BV
- Price to cash flow ratio – P/CF
- Price to sales ratio – P/S

Introductory Comments

In a previous chapter we analyzed the stocks' basics. These concepts are essential in comprehending how stocks are valued. This chapter contains two sections. The first section presents the basics of stock valuation. The second section analyzes how stocks are valued. This section has five sub-sections. The first sub-section describes the dividend discount model. The second sub-section presents the price-to-earnings (P/E) ratio. The third sub-section examines the factors which affect the dividend growth rate. The fourth sub-section analyzes the factors that affect the expected earnings per share. Finally, the fifth sub-section presents three other ratios used to calculate the value of the stock.

5.1 STOCK VALUATION BASICS

The net earnings of a firm can be either distributed to its shareholders as dividend or retained by the firm. If earnings are distributed as dividend, the current income of shareholders increases. If the firm retains its earnings, then these earnings will finance the new investments. If the firm has a positive return from these additional investments, the total earnings of the firm will increase and consequently the stock price will increase too. Thus, in the second case shareholders will have capital gains, most of which will be realized in the future. According to the above analysis, it is obvious that the dividend payout ratio (or the retention rate) affects directly the shareholders' return. The **dividend payout ratio** is the percentage of the firm's annual net earnings paid to the shareholders in the form of cash dividends. For example, if the dividend per share is €10 and the earnings per share are €25, then the dividend payout ratio is 40%. The supplement of the dividend payout ratio (i.e. 1.00 – dividend payout ratio), is the **retention rate**. The retention rate is the percentage of the annual net earnings that are not paid to shareholders but instead are retained by the firm to finance its investment projects.

We have already mentioned that a security's total return consists of two parts: yield and capital gain (or loss). If the security is a stock, its total return is constituted by the dividend yield and the capital gain (or loss). The dividend yield is the current dividend paid on a share of common stock, expressed as a percentage of the current market price of the firm's common stock. At this point, we should notice that most firms follow a stable dividend policy. In the past, a stable dividend policy meant that the firm should pay the same dividend every year. Today, most investors expect from the firms that follow the above policy to pay a dividend which will be increased at a relatively constant growth rate over time (i.e. from year to year). This constant dividend growth rate is based on the expected growth of the firm (and its earnings) and the expected inflation. Capital gain is the amount by which the sale price of a stock exceeds the purchase price of that stock. Capital loss is the amount by which the sale price of a stock is less than its purchase price. Note that in various studies conducted for the US market, it was found that the common stock's dividend yield had, on average, the same effect on the total common stock's rate of return, as its (i.e. common stock's) capital gains. According to the above, it is obvious that the investors pay attention to the stock's dividend yield. This happens, because many investors wish to have current income from their investment, to be able to plan better their expenses.

5.2 STOCK VALUATION

In the previous chapter we describe "valuation" as the process of determining the market value of a security. As a consequence, stock valuation determines the stock's intrinsic value. There are two fundamental approaches in common stock valuation:

- The present value approach
- The earnings multiplier approach (the price-to-earnings ratio).

5.2.1 The present value approach

The stock's purchaser pays an amount of money today in order to receive cash flows in the future. This amount of money, which is the stock's market price, is equal to the present value of the future cash flows that the purchaser will receive. In order to determine the stock's present value, its future cash flows should be discounted using an appropriate rate. The most straightforward measure of cash flows is dividend, because they go directly to the investor. Thus, the present value of the total future dividends is the intrinsic value of the stock. Consequently, according to this method of valuation, which is often known as **dividend discount model,** the intrinsic value of a share of common stock is equal to the discounted value of the dividends forecast to be paid on the stock:

$$IV = \frac{D_1}{(1+k)^1} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + ... + \frac{D_\infty}{(1+k)^\infty} \quad (5.1)$$

where, IV = the intrinsic value of the stock, D_1, D_2,... = the annual dividend expected to be received at the end of the first, second, etc year, and k = the required rate of return applicable for an investment with this degree of risk.

The discount rate used in the above equation, is the investors' required rate of return on that particular stock. Thus, the required rate of return is unique for each stock and is determined by the risk of that stock. Furthermore, this rate of return is the shareholder's opportunity cost, because it represents the rate of return of the best alternative available investment of a comparable risk, which the investor leaves behind. Thus, market rates also affect the discount rate used to determine the intrinsic value of the stock.

The above approach appears to neglect capital gains, which are profits generated by an increase in the price of the stock. Many investors are primarily interested in capital gains when they purchase stocks. This view is incorrect, because we assume explicitly in equation (5.1) that capital gains (as reflected in the expected sales price) are part of the stock's value. Put differently, expected price in the future is built into the dividend discount model given by equation (5.1) - it is simply not visible. Our point is that the price at which you can sell a stock in the future depends upon dividend forecasts at this time.

The previous formula can be used to determine the intrinsic value of the stock only if we are able to forecast all future dividends. Because a common stock does not have a fixed lifetime, a virtually infinite stream of dividend payments must be forecast. Although this may seem to be an impossible task, with the addition of certain simplifying assumptions we can make the task viable. These simplifying assumptions center on dividend growth rates. In this case, we have three simple models:

(a) The **constant growth or perpetual growth model**[1]. According to this model, we assume that the future dividends will grow at a constant rate (g) each year for an infinite period. In this case, the previous equation (5.1) can be written as follows:

$$IV = \frac{D_0(1+g)^1}{(1+k)^1} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \ldots + \frac{D_0(1+g)^\infty}{(1+k)^\infty} \qquad (5.2)$$

where, D_0 = the current (last) annual dividend per share.

If we multiply both sides of equation (5.2) by [(1 + k)/(1 + c)] and subtract equation (5.2) from the result, we have:

$$\frac{IV(1+k)}{1+g} - IV = D_0 - \frac{D_0(1+g)^\infty}{(1+k)^\infty}$$

Assuming that k is greater than g, the second term of the right hand side of the equation will be zero. In this case, the previous equation will be[2]:

$$IV\left[\frac{1+k}{1+g} - 1\right] = D_0 \Rightarrow$$

$$IV\left[\frac{(1+k)-(1+g)}{1+g}\right] = D_0 \Rightarrow$$

$$IV(k-g) = D_0(1+g) \Rightarrow$$

$$IV = \frac{D_1}{k-g} \qquad (5.3)$$

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[1] *This model is also known as the Gordon – Shapiro model, from the names of Myron I. Gordon and E. Shapiro.*

[2] *If k<g, the model provides meaningless results, because the denominator becomes negative. In case we have to find the stock's intrinsic value, we can apply the multiple-growth model, presented later in this chapter.*

Note that D_1 is the dividend expected to be received at the end of year 1 and is calculated by multiplying the dividend payment in the current period (D_0) by (1 + g); that is, $D_1 = [(D_0)\times(1 + g)]$. This method is used quite often in stock valuation, because it is particularly simple and describes the actual behavior of a large number of companies. Notice that as the required rate of return (k) is greater than the growth dividend ratio (g), the present value of each future dividend payment decreases as we move further to the future. Thus, later dividends have rather insignificant effect in the intrinsic value of the stock.

(b) **The zero - growth model**. According to this model, we assume that the dividend growth rate will be zero (g = 0). In this case, the dividend discount model reduces to a perpetuity and equation (5.1) can be written as follows:

$$IV = \frac{D}{(1+k)^1} + \frac{D}{(1+k)^2} + \frac{D}{(1+k)^3} + \ldots + \frac{D}{(1+k)^\infty} = \frac{D}{k} \qquad (5.4)$$

where, D = the constant annual dividend payment expected for all future time periods.

As noted before this model make the following assumptions:

- dividends grow at a constant rate,
- there is an infinite time horizon, and
- the required rate of return on the investment (k) is greater than the infinite expected growth rate (g).

The most obvious case in which the above simplified valuation procedure cannot be used is that of a rapidly growing company. High-growth companies can be characterized as growing at quite rapid rates relative to general economic growth. Quantitatively, these companies would be classified as growing at rates of 15-30%, compared with the growth rates of 8-9% that was shown to be representative of "non growth" companies. At the same time we cannot assume that this rapid growth will remain constant, as we should anticipate a transition to a more normal pace which will be in line with the general economic growth. To deal with these situations, we can use the multiple-growth model.

(c) **The multiple - growth model**. According to this model, we assume that there are different dividend growth periods (usually two or three). For example, the two-period model assumes that the specific firm has two distinct growth periods, each one leading to different dividend policies. The first period lasts usually 2 to 10 years and is characterized by a dividend growth rate, higher (or lower) than the normal. The second period lasts thereafter and is characterized by a normal dividend growth rate in line with the general corporate average. In this case, the present value of the dividends for the first period is calculated and for the second period the constant growth model can be applied. The same process can also be followed for more than two periods. The three – period model assumes that all firms go through three phases, analogous to the concept of the product life cycle. In the growth phase, usually over a 5 year period, a firm experiences rapid earnings growth as it produces new products and expands market share. In the transition phase, that may occur over a period ranging from 5 – 20 years, the firm's earnings begin to mature and decelerate to the rate of growth of the economy as a whole. In the maturity phase all firms are assumed to reach a steady state of growth in line with the overall economy.

The dividend discount model determines the intrinsic value of the stock. Traditionally, investors and analysts compare the stock's intrinsic value with the current market price. If the stock's intrinsic value is higher than the current market price, the stock is undervalued by the Stock Market and thus it should be purchased (or held if already owned). If the stock's intrinsic value is lower than the current market price, then this stock is overvalued and it should be avoided, sold if held, or possibly sold short. If the stock's intrinsic value is equal to its current market price, then the market has valued the stock correctly. According to the above, it is obvious that the stock's intrinsic value, determined by using the dividend discount model, is the correct market price. The problem with the intrinsic value is that all investors do not have the same required rate of return and the same expectations for the future growth in dividends. Thus, a stock may have several intrinsic values, depending on who makes the valuation. This is the reason why several investors want to buy a stock at a specific price, while some others want to sell a stock at the same price. Thus, the stock price in the market reflects the opinion about the real value of the stock that the majority of the investors has.

Example 1

ABC is currently paying an annual dividend of €4 per share, which is not expected to change. Investors require a rate of return of 16% to invest in a stock with the riskiness of ABC. Calculate the intrinsic stock value of ABC.

Answer:

The stock's intrinsic value is calculated using the zero growth model:

$$IV = \frac{D}{k} + \frac{4}{0.16} = 25$$

Thus, the stock price of ABC is €25.

Example 2

ABC currently pays a dividend of €3 per share on its common stock. The dividend is expected to grow at 8% per year forever. Stocks with similar risk currently are priced to provide a 17% expected return. What is the intrinsic value of ABC stock?

Answer:

Because dividends are expected to grow each year at a constant rate, we will use the constant growth model. In this case, the intrinsic value of ABC is €36, which was found as follows:

$$IV = \frac{D_1}{k-g} + \frac{3(1+0.08)}{0.17-0.08} = 25$$

Example 3

ABC has been undergoing rapid growth for the last few years. The current dividend of €2 per share is expected to continue to grow at the rapid rate of 20% a year for the next three years. After that time ABC is expected to slow down, with the dividend growing at a more normal rate of 8% a year for the indefinite future. Because of the risk involved in such a rapid growth, the required rate of return on this stock is 18%. Calculate the intrinsic value of ABC stock.

Answer:

We calculate the dividends in each year of supernormal growth (i.e. the dividends of the first three years):

$D_1 = D_0 \times (1+g_1) = 2 \times (1+0.20) = 2.40$

$D_2 = D_1 \times (1 + g_1) = D_0 \times (1 + g_1)^2 = 2 \times (1+0.20)^2 = 2.88$

$D_3 = D_2 \times (1 + g_1) = D_1 \times (1 + g_1)^2 = D_0 \times (1 + g_1)^3 = 2 \times (1+0.20)^3 = 3.46$

We discount each of the above dividends at the required rate of return, which is 18%. Summing the three discounted dividends produces the value of the stock for its first three years only, which is €6.21.

$$2.40 \times (0.8475) = 2.03$$
$$2.88 \times (0.7182) = 2.07$$
$$3.46 \times (0.6086) = 2.11$$

Present value of the first three years of dividends = (2.03 + 2.07 + 2.11 =) 6.21.

To evaluate years 4 on, when constant growth is expected, the constant growth model is used.

$$P_3 = \frac{D_4}{k-g_2} + \frac{3.46(1+0.08)}{0.18-0.08} \approx 37.37$$

Thus, €37.37 is the expected price of the stock at the beginning of year 4 (end of year 3). It must be discounted back to the present, using the present value factor for three years and 18%, 0.6086. Therefore, the present value of P_3 at time period zero, representing the discounted value of dividends from year 4 to ∞ is (37.37×0.6086 =) €22.74. Adding this value to the present value of all dividends to be received during the abnormal growth periods produces the intrinsic value of the ABC stock, which is (6.21+22.74 =) €28.95.

5.2.2 The earnings multiplier approach

The price-to-earnings (P/E) ratio or **earnings multiplier** is calculated as the current stock price divided by earnings per share (typically represented by the latest 12-month reported earnings per share). However, security analysts sometime use estimated earnings per share for the next 12 months. This ratio indicates how much the market as a whole is willing to pay per euro of reported earnings. It is standard investing practice to refer to stocks as selling at, say, 15 times earnings. That is why the P/E ratio is also referred to as the earnings multiplier. Furthermore, the P/E ratio shows the years required for the investor to recover the money he/she paid to purchase the stock, assuming that earnings per share remain constant over time. Low P/E ratios are typically associated with low earnings growth and cyclical business, while high P/E ratios are typically associated with high earnings growth and non-cyclical business. Generally, if the P/E ratio of a stock is higher than the P/E ratio of the market, or the corresponding industry, then either this firm is actually among the best in the industry, or investors have overestimated its earnings and the firm is overvalued. On the contrary, a low P/E ratio implies that either the firm is not preferred by the investors (perhaps because the management is not efficient or the firm's prospects are not so good), or it is undervalued by the market (because investors have underestimated its growth prospects).

Many security analysts prefer to estimate the value of common stocks using the P/E ratio approach rather than the discount dividend model. Stock valuation using the P/E ratio approach is based on the following computation:

$$IV = P_0 \equiv E_0 \times [P_0 / E_0]$$

To find the stock's intrinsic value we estimate the earnings per share of the next year (E_1) and multiply them with an estimate of the "normal" P/E ratio[3]. For example, if investors are willing to pay 12 times expected earnings, they would value a stock they expect to earn €4 a share during the following year at €48. The problem is which P/E ratio will be used. The "normal" P/E ratio may differ from the P/E that prevails in the market. The current P/E ratio indicates the prevailing attitude of investors toward a stock's valuation. However the analyst using this approach must decide if he/she agrees or disagrees with the prevailing P/E ratio.

What determines a P/E ratio? The factors that determine the value of the P/E ratio can be shown by employing the dividend discount model. Under the assumption of constant growth the stock price will be:

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[3] *Instead of the expected earnings per share of the next year, several analysts use the normalized earnings per share of the evaluated company, defined as the firm's earnings in its usual operating activities. In other words, we adjust the earnings of the firm to what they would be at the midpoint of an economic cycle. The purpose of the normalization procedure is to abstract the earning power of the firm from abnormal economic influences such as recession or boom.*

$$P_E = \frac{D_1}{k-g}$$

where, P_E = the stock's estimated price from the model. If we divide both sides of the above equation with the expected earnings during the next 12 months E_1, we obtain:

$$\frac{P_E}{E_1} = \frac{D_1/E_1}{k-g} = \frac{1-b_1}{k-g} \qquad (5.5)$$

where, (D_1/E_1) = the expected dividend payout ratio of the next year, b_1 = the expected retention rate for the next year (considered as constant, assuming that the examined firm maintains a constant rate of retained earnings each year).

From the above, it is obvious that the P/E ratio depends on the following factors:

- the firm's expected dividend payout ratio (or its retention rate),
- the required rate of return on the firm's stock (which is related to the market interest rates), and
- the expected growth rate of dividends.

The most important factors in the preceding determination of the P/E ratio are the required rate of return and the growth rate of dividends. Even a small change in one of these two factors, will have a large effect on the P/E ratio. At this point, we should note that the above three factors usually affect each other. For example, an increase in a firm's dividend payout ratio will increase its P/E ratio, other things being equal. However, such an action would in all likelihood reduce the firm's investments, and consequently, its future earnings. The decrease in the firm's future earnings will cause a decrease in the growth rate of dividends. The latter will decrease the firm's P/E ratio, offsetting the positive effect of the increased dividend payout ratio.

Example 4

The expected dividend payout ratio of the ABC firm is 60%. The stock's earnings and dividends are expected to grow at 9% per year indefinitely. The latest 12 – month earnings per share were €5. The required return on stocks with similar risk is 14%. Calculate the intrinsic value of ABC stock, using the P/E approach.

Answer:

The P/E ratio of the firm ABC is:

$$P/E = (0.60)/(0.14\text{-}0.09) = 12$$

The firm's expected earnings per share for the next year will be:

$$E_1 = E_0 \times (1+g) = 5 \times (1+0.09) = €5.45$$

Therefore, the stock's ABC intrinsic value will be:

$$P_0 = E_1 \times [P/E] = (5.45) \times (12) = 65.40.$$

5.2.3 Factors that affect the dividend growth rate

So far, the analysis has assumed that the growth rate of dividends and earnings (g) is known[4]. But what determines this growth rate? The growth rate of earnings or dividends depends mainly on the following two factors[5]: (1) the retention rate, which is the proportion of earnings retained and reinvested by the firm, and (2) the rate of return earned on investment, which is the return on equity (ROE). More specifically, the growth rate of earnings and dividend (g) of a firm, with no external financing, is:

$$g = b \times ROE \qquad (5.6)$$

where, b = the retention rate (or 1.00 – dividend payout ratio), and ROE = the return on equity. For example, if a firm retains 60% of its annual net earnings (and, consequently, distributes 40%) and invests these funds in projects, with a ROE of 15%, then its net earnings will grow at 9% per year:

$$g = b \times ROE = (0.60) \times (0.15) = 0.09$$

To estimate the growth rate of dividends, therefore, it is necessary to estimate the retention rate (or the dividend payout ratio) and the return on equity. Because the valuation model is a long run model, we should estimate only relatively permanent changes, although short run changes may affect expectations. Payout ratios for most firms vary over time, but reasonable estimates can often be obtained for a particular firm. From the Financial Management we know that the return on equity can be broken down into various components, using the Du Pont analysis, as follows:

$$ROE = \frac{\text{Net Income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total Assets}} \times \frac{\text{Total Assets}}{\text{Equity}}$$

$$ROE = \text{Net Profit Margin} \times \text{Total Assets Turnover} \times \text{Equity Multiplier}$$

From the above it follows that when an analyst values a company, he can estimate each of the above three ratios that determine the firm's future return on equity (using past data and his expectations for the firm's future prospects). By multiplying these three component ratios he gets an estimate of the firm's future return on equity. This estimated return on equity, multiplied by the firm's retention rate, will provide an estimate for the earnings growth rate (and consequently the dividend growth rate) of the examined firm.

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[4] *The growth rate of dividends may differ in the short run from the growth rate of earnings, but it is not expected to differ in the long run. For this reason most security analysts assume that these two rates are equal when they value stocks. In fact, they assume a relatively constant dividend payout ratio (dividend per share over earnings per share), so the growth of dividends is dependent on the growth in earnings.*
[5] *It also depends on financial leverage. In this case, we assume that there is no external financing.*

The previous analysis showed how an analyst can estimate the future dividend growth rate (g) of the firm. An alternative approach is to predict the future growth rate of dividends based upon the growth rate of dividends that the firm had in the past. However, in both cases the important variable that should be predicted remains the same; that is, the change in earnings over some future period. The problem in the second approach, as we will see later in this chapter, is that past growth rates of earnings are not good estimates for future growth rates of earnings. Empirical studies have shown that most firms' growth rates of earnings vary over time. Because of the lack of consistency in earnings trends, interested investors can estimate the expected growth rate in earnings, or they can use earnings per share forecasts, provided either mechanically (i.e., by a time series model) or by security analysts.

5.2.4 Factors that affect the expected earnings per share

The present value approach and the earnings multiplier approach use the expected earnings per share and the required rate of return of the examined firm. Which are the factors that affect these two variables and how can an investor estimate them? We know from Financial Management that the firm's annual earnings per share (EPS), are determined by two factors:

$$\text{EPS} = \frac{\text{Net Income}}{\text{Equity}} \times \frac{\text{Equity}}{\text{Shares Outstanding}}$$

$$\text{EPS} = \text{Return on equity (ROE)} \times \text{Book value per share}$$

From the above it follows that an analyst, or an investor who is interested in a company, can make estimates for each of the two above component ratios that determine a company's future earnings per share (using past data and his expectations for the company's future prospects). By multiplying these two ratios he/she estimates the company's expected earnings per share. In the U.S.A. there are various financial organizations (i.e., the Value Line) providing forecasts (based on their analyses) for most firms' future earnings.

Alternatively, an analyst (or an investor) can forecast the firm's future earnings by using several well known extrapolative statistical models (i.e. the simple line trend model, the simple exponential model, the simple autoregressive model etc.). These models apply a formula to historical data and project results for a future period. By choosing a statistical model that best fits the data, an analyst, in effect, says that the past pattern of earnings will predict the future. Earlier empirical studies found that statistical methods provide more accurate forecasts than the analysts. More recent studies found the opposite. The major finding of the vast amount of literature to date is that analysts do a better job than simple extrapolative models. On the other hand, the analysts generally fail to outperform company management forecasts.

Estimating the required rate of return on an investment is a difficult task. Recall from the chapter "Return and Risk" that the required return is the minimum rate of return that the investors require from an investment in order to undertake it. The required rate of return on a stock contains three components:

- The real risk-free rate, which is the compensation that an investor requires in order to postpone his/her current consumption (pure time value of money). This is the basic interest rate, assuming no inflation and no uncertainty about future flows, and is based on the long run real growth rate of the economy, since the invested capital should grow at least as fast as the economy.
- The expected rate of inflation.
- The risk premium (i.e. a compensation that investors demand for an investment's uncertainty), which depends on various sources of uncertainty such as business risk, financial risk, liquidity risk, exchange rate risk and country (political) risk.

Thus, if an investor (or an analyst) wants to forecast the required rate of return on a stock, he/she can follow these steps[6]:

First, the investor will calculate the expected rate of inflation [E(I)] and the real risk-free rate (RRFR). Then, he/she will calculate the nominal risk-free rate (NRFR) from the following equation:

$$NRFR = [1+RRFR][1+E(I)] - 1.$$

Second, the investor will calculate a risk premium, which is the additional compensation demanded by the investor before purchasing a risky asset such as a common stock. In chapter 2 we noted that investors demand a risk premium because of the uncertainty of returns expected from an investment. In this particular case the investor should examine the following risks that affect the stock that he/she is interested in.

The business risk, which is the uncertainty of income flows that is caused by the nature of the firm's business. It can be measured by the variability of the firm's operating income over time. Sales variability is the prime determinant of earnings variability. In turn, an investor can obtain an indication of a firm's business risk by calculating the coefficient of variation of the firm's sales during the most recent 5 to 10 years, and the standard deviation of the annual percentage change in sales during the same period, and compares both ratios with the corresponding industry ratios.

The financial risk, which is the additional uncertainty of returns to equity holders due to the introduction of debt into the capital structure of the firm. An investor can obtain an indication of a firm's financial risk, by calculating some of the well known leverage ratios (i.e. the debt-to-equity ratio) and compares them with the corresponding industry ratios.

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[6] *This calculation of the required rate of return is based on the traditional approach of the sources of investment risk. Alternatively, the required rate of return can be calculated by using the Capital Asset Pricing Model (CAPM), which will be discussed in a subsequent chapter.*

The liquidity risk, which is the uncertainty introduced by the secondary market for the investment. The most common liquidity indicators are the number of the firm's shareholders, the number of the shares outstanding, and the stock's trade volume on the exchange. Information about the liquidity of the firm can be produced by comparing the above indicators with the corresponding industry indicators.

The exchange risk and the political risk. If the firm operates in both domestic and foreign markets, the investor should also take into account the risk of a change in the exchange rate between the foreign currency and the euro, as well as the risk of a significant change in the political or economic environment of the foreign country.

Furthermore, the investor can calculate the beta coefficient of the stock, which is a measure of the systematic risk of the stock, and he/she can compare it to the beta coefficient of the corresponding industry[7]. Recall that systematic risk is the variability of returns that is due to macroeconomic factors that affect all risky assets. Because it affects all risky assets, it cannot be eliminated by diversification.

Following the above analysis, the investor will be able to find out if the examined firm has higher or lower total risk than that of the industry or the market as a whole. In consequence, the risk premium demanded by investors for purchasing the common stock of the firm should be proportional to the undertaken risk. Note that the risk premium changes over time. Investor pessimism will increase the risk premium and the required rate of return, while investor optimism lowers both.

In the U.S.A., the Ibbotson Associates[8] using past data for 1926-2001, estimated that the average annual rate of return on the common stocks exceeded the Treasury bills' average annual rate of return by 9.1% (using the arithmetic mean of annual returns) and by 7.5% (using the geometric mean of annual returns). Since 1988 the Ibbotson Associates recommend the use of the rate of return on mid – term Treasury bonds (instead of the Treasury bills) as a risk free security. This replacement is based on the belief that most investors have an investment time horizon longer than the Treasury bills maturity. More specifically, investors usually have a medium-term investment time horizon of 5-10 years. In this case, the Ibbotson Associates' risk premium (computed as the stock return less the return on intermediate government bonds) is 8.2% (if the arithmetic mean of annual returns is used) and 6.6% (if the geometric mean of annual returns is used). As we have already mentioned in the chapter "Return and Risk", the geometric mean is appropriate for long-run asset class comparisons, whereas the arithmetic mean should be used when we want to estimate the risk premium for a given year (i.e. the expected rate of return for the next year). In case we are interested in the application of the dividend discount model, which has a long-term horizon, the geometric mean is the most suitable measure. Thus, the long-term historical risk

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[7] *In fact, the beta coefficient is a standardized measure of systematic risk, based upon a stock's covariance with the market portfolio. We will discuss the beta coefficient and its calculation in subsequent chapter.*
[8] *Ibbotson Associates, (annual), Stocks, Bonds, Bills and Inflation, Ibbotson Associates, Chicago, IL.*

premium to use in the U.S.A. should be about 6.6%. However, note that the above estimate of the risk premium in the U.S.A. does not remain constant over time.

5.2.5 Other valuation techniques

Analysts often use several other ratios as indicators of relative value of stocks. These ratios do not have the theoretical background that P/E ratio has, but provide additional information about the relative value of a stock. They are called relative valuation techniques because, when performing company stock analysis, analysts compare these ratios to similar ratios for the aggregate market, other industries, and other stocks in the industry. We should point out, however, that most analysts consider the analysis with these ratios as additional to the analysis of the P/E ratio. The most important ratios of this category are the following:

Price-to-book value ratio (P/BV) or market-to-book ratio. This ratio is calculated by dividing the market price of the stock by the stock book value per share. The book value of a firm is the accounting value of the equity as shown on the books (i.e., balance sheet). It is the sum of common stock outstanding, capital in excess of par value, and retained earnings. Dividing the total book value by the number of common shares outstanding produces the book value per share. The P/BV ratio indicates what the market is willing to pay for a firm's book value. In effect, it shows if a stock is overvalued or undervalued regarding its book value. We should point out, however, that this ratio should be used carefully, because while the stock book value is given in historical prices, the price of the stock is determined by the market. The P/BV ratio is often used to value companies, particularly banks. This is because the assets of banks have book values that are similar to their market values. It is usual for companies with relatively high return on equity to exhibit high relative P/BV ratios, and vice versa.

Common stocks are often divided into two categories: growth stocks and value stocks. In general, growth stocks are stocks of companies that have experienced rapid increase in earnings, while value stocks are stocks whose market price seems to low relative to measures of their worth. The most commonly used measure for classifying stocks as growth or value is the P/BV ratio. A P/BV ratio that is high relative to the universe of stocks is an indicator of growth, while a low relative P/BV ratio is an indicator of value.

Many analysts believe that stocks with low P/BV ratio will have higher returns than stocks with high P/BV ratio. Recent empirical studies found a significant negative relationship between this ratio and average stock returns[9]. In consequence, this ratio gained in popularity and credibility as a relative valuation technique. Thus, investors should examine this ratio over time[10] and compare it to similar ratios for the

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[9] *For more information see Rosenberg, Reid and Lanstein (1985), and Fama and French (1992).*
[10] *It is obvious that the P/BV ratio varies over time, as the firm's stock price fluctuates.*

aggregate market, other industries, and other stocks in the industry. In general, if the P/BV ratio is less than 1, the stock is considered very attractive, while it still is a good buy if the ratio is close to 3. As far as the factors that determine the size of the P/BV ratio, it is a function of return on equity relative to the company's cost of equity. If the company earned a return on capital (which for the equity holder is the return on equity) equal to its cost of funds this ratio would be 1. In contrast, if the return on equity is much larger, it is a growth company and investors are willing to pay a premium over book value for the stock. Note that to properly implement this technique investors should also take into account the relative growth rate of the firm's equity, and the firm's risk.

Price-to-cash flow (P/CF) ratio. This ratio is calculated by dividing the market price of the stock by its cash flows per share. The specific cash flow measure is typically EBITDA, which is defined as earnings before interest, taxes, depreciation, and amortization. Some analysts use operating cash flow when calculating this ratio, whereas others prefer free cash flow (that is, operating cash flow net of new investment). Some analysts employ this ratio as a measure of relative value, because they argue that cash flow values are generally less prone to manipulation than earnings, which can be affected by the company's choice of accounting practices, and thus are commonly viewed as subject to manipulation. Several analysts claim that stocks with low P/CF ratio have higher rates of return than stocks with high P/CF ratio. Note that to implement this technique it is essential to compare the P/CF ratio of the examined firm to similar ratios for the aggregate market, other industries, and other stocks in the industry, taking into account the relative growth rate of the firm's cash flows and the firm's risk.

Price-to-sales (P/S) ratio. This ratio is calculated by dividing the firm's total market value (that is, its share price times the number of shares outstanding[11]) by its total sales. In other words, this ratio shows what the market (i.e. the investors) is willing to pay for a firm's revenues. Several analysts claim that firms with low P/S ratio have higher rates of return than firms with high P/S ratio. The P/S ratio should also be compared to the industry similar ratio, considering also the relative growth rate of the firm's sales, and the firm's risk.

Activity 1/Chapter 5

Four years ago, firm ABC paid a dividend of €0.8 per share. ABC is currently selling for €30 per share and paying a dividend of €1.6 per share. The firm during the past years followed a constant growth dividend policy, and is expected to follow the same dividend policy for the next five years. After these five years, the expected growth rate in dividends will be 8% per year for the foreseeable future. If the require rate of return is 18%, would you purchase this stock?

[11] This product is also called "market capitalization" of the firm, and is equal to the market value of the firm's outstanding equity.

Synopsis

- **The dividend yield is the current dividend paid on a share of common stock, expressed as a percentage of the current market price of the firm's common stock.**
- **"Valuation" is the process of determining the securities' market value. Thus valuation leads to the calculation of the stock's intrinsic value.**
- **Valuation is the process of determining the market value of a security. Thus, stock valuation determines the stock's intrinsic value.**
- **There are two fundamental approaches in common stock valuation: the present value approach, and the earnings multiplier approach.**
- **According to the present value approach, which is often known as dividend discount model, the intrinsic value of a share of common stock is equal to the discounted value of the dividends forecast to be paid on the stock. This approach can be employed only if we are able to forecast all future dividends. Although this may seem to be an impossible task, with the addition of certain simplifying assumptions we can make the task viable. These simplifying assumptions center on dividend growth rates.**
- **Assuming that the future dividends will grow at a constant rate (g) each year for an infinite period, then the stock's intrinsic value is equal to the expected dividend of the next year (D_1) divided by the difference between the required rate of return (k) and the expected growth rate in dividends (g).**
- **Assuming that future dividends remain constant for the foreseeable future, then the stock's intrinsic value is equal to the constant dividend expected for all future time periods (D) divided by the required rate of return (k).**
- **The price-to-earnings (P/E) ratio or earnings multiplier is calculated as the current stock price divided by earnings per share (typically represented by the latest 12-month reported earnings per share). However, security analysts sometime use estimated earnings per share for the next 12 months. To find the stock's intrinsic value we estimate the earnings per share of the next year (E_1) and multiply them with an estimate of the "normal" P/E ratio.**
- **The P/E ratio depends on the firm's expected dividend payout ratio (or its retention rate), the required rate of return on the firm's stock (which is related to the market interest rates), and the expected growth rate of dividends.**
- **The dividend growth rate (g) depends on the retention rate (b) and the return on equity (ROE).**
- **The annual earnings per share (EPS) of a company depend on the return on equity and the book value per share.**
- **The required rate of return on a stock contains three components: the real risk-free rate (which is based on the long run real growth rate of the economy), the expected rate of inflation, and the risk premium (which depends on various sources of uncertainty such as business risk, financial risk, liquidity risk, exchange rate risk and country or political risk).**

Answer to the Activity

Activity 1

Answer:

The annual constant dividend growth rate of ABC for the last four years is 20%, and is found as follows:

$$D_0 = D_{-4}\times(1 + g_1)^4 \Rightarrow (1 + g_1)^4 = (D_0 / D_{-4}) \Rightarrow (1 + g_1)^4 = (1.66/0.8) \Rightarrow$$

$$(1 + g_1)^4 = (2.0750) \Rightarrow (1 + g_1) = (2.0750)^{1/4} \Rightarrow (1 + g_1) = 1.2002 \Rightarrow$$

$$g_1 = 0.2002 \text{ or } g_1 \approx 20\%$$

The dividends in each year for the next five years will be:

$$D_1 = D_0\times(1+g_1) = 1.66\times(1+0.20) \approx 1.99$$

$$D_2 = D_0\times(1 + g_1)^2 = 1.66\times(1+0.20)^2 \approx 2.39$$

$$D_3 = D_0\times(1 + g_1)^3 = 1.66\times(1+0.20)^3 \approx 2.87$$

$$D_4 = D_0\times(1 + g_1)^4 = 1.66\times(1+0.20)^4 \approx 3.44$$

$$D_5 = D_0\times(1 + g_1)^5 = 1.66\times(1+0.20)^5 \approx 4.13$$

We discount each of the above dividends at the required rate of return, which is 18%. Summing the five discounted dividends produces the value of the stock for its first five years only, which is €8.73.

$$1.99\times(0.8475) \approx 1.69$$

$$2.39\times(0.7182) \approx 1.72$$

$$2.87\times(0.6086) \approx 1.75$$

$$3.44\times(0.5158) \approx 1.77$$

$$4.13\times(0.4371) \approx 1.80$$

Present value of the first five years of dividends = (1.69+1.72+1.75+1.77+1.80 =) €8.73. To evaluate years 6 on, when constant growth is expected, the constant growth model is used.

$$P_5 = \frac{D_6}{k - g_2} = \frac{4.13(1+0.08)}{0.18-0.08} \approx 44.6$$

Thus, €44.6 is the expected price of the stock at the beginning of year 6 (end of year 5). It must be discounted back to the present, using the present value factor for five years and 18%, 0.4371. Therefore, the present value of P_5 at time period zero, representing the discounted value of dividends from year 6 to ∞ is (44.6×0.4371 ≈) €19.49. Adding this value to the present value of all dividends to be received during the first five year-period produces the intrinsic value of the ABC stock, which is (8.73+19.49 =) €28.22. Thus, the stock's intrinsic value (€28.22) is lower than the current market price (€30), and consequently this stock is overvalued and it should be avoided.

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CHAPTER 6

PORTFOLIO THEORY

The Scope of the Chapter

In this chapter, we help you understand the basics of portfolio theory, which is necessary for constructing an optimal portfolio of assets.

Learning Objectives

When you have finished studying this chapter, you will be able to:

- list the basic assumptions underling portfolio theory
- calculate the expected rate of return and risk of a single asset and a portfolio of assets
- explain the importance of the correlation and covariance of two assets in measuring a portfolio's risk
- describe what is meant by an efficient portfolio
- explain what is meant by a Markowitz efficient frontier
- estimate a security's and a portfolio's systematic and unsystematic risk
- show how the unsystematic risk can be eliminated, when more securities are added to a portfolio
- describe the concept of an optimal portfolio and how it is selected from all the portfolios available on the Markowitz efficient frontier
- define the characteristic line and the beta coefficient.

Key Words

- Covariance
- Correlation coefficient
- Minimum variance portfolio
- Attainable set of portfolios
- Efficient portfolio
- Efficient frontier
- Optimal portfolio
- Characteristic line
- Beta coefficient
- Unsystematic (or diversifiable) risk
- Systematic (or non-diversifiable) risk
- Diversification

Introductory Comments

One of the most important theories developed in finance during the past decades is the portfolio theory. According to this theory, the investor must consider the relationships among the investments if he/she wants to construct an optimum portfolio that will meet his/her needs and objectives. Specifically, it deals with the selection of portfolios that maximize expected returns consistent with individually accepted levels of risk. This chapter contains six sections. The first section presents how the expected return and risk of a portfolio is estimated. The second section describes how efficient portfolios are determined. The third section describes how an investor chooses an optimal portfolio. The fourth section analyzes the single-index model. The fifth section describes how the single-index model is used. Finally, the sixth section examines the concept of systematic and unsystematic portfolio risk.

6.1 EXPECTED RETURN AND RISK OF A PORTFOLIO

Portfolio is a collection of assets held by a single investor, whether an individual or institution. **Portfolio theory** is concerned with an investor's portfolio, i.e. the combination of assets invested in and held by an investor. The basic portfolio theory was developed by Harry Markowitz[1] and is based on specific characteristics of securities[2]. These characteristics are the expected rate of return on each security, its expected risk (measured by the standard deviation of the security's returns) and the covariance between the return of securities.

In chapter "Return and risk", we mentioned that the expected rate of return on a security is the weighted average of all the security's possible returns, where the weights are the probabilities associated with the returns. That is:

$$\mathrm{E}(r) = \sum_{i=1}^{n} P_i r_i \qquad (6.1)$$

where, E(r) = the expected return from the security, r_i = the i^{th} possible return, P_i = the probability of the i^{th} possible return occurring (and $\sum P_i = 1$), and n = the number of possible returns.

In the same chapter, we defined risk the variability of possible outcomes around their expected value or their mean. Recall that the dispersion of possible returns about the expected return is measured by the variance (and equivalently, the standard deviation) of the probability distribution and can be used as a proxy of risk. The variance of returns is calculated as the weighted average of the squared deviations from the expected rate of return, where the weights are the probabilities associated with the returns. The standard deviation of returns can be calculated as follows:

$$\sigma = \left\{ \sum_{i=1}^{n} P_i \left[r_i - \mathrm{E}(r) \right]^2 \right\}^{\frac{1}{2}} \qquad (6.2)$$

where, σ = the standard deviation of returns, r_i = the i^{th} possible return, P_i = the probability of the i^{th} possible return occurring, E(r) = the security's expected return, and n = the number of possible returns.

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[1] *See Markowitz (1952), and Markowitz (1959). Note that H. Markowitz (together with W. Sharpe and M. Miller), won Nobel prizes in economics in 1990.*

[2] *Portfolio theory encompasses the investor's entire set of assets, real and financial. In this chapter, however, we are concentrating on financial assets, so that this theory can be more easily comprehensible.*

The variance (Var) is:

$$\text{Var} = \sigma^2$$

The variance (or the standard deviation) is thus a quantitative description of risk. Moreover, this measure of risk is simply a proxy or surrogate for risk, since other measures could be used. The problem with using the variance as a measure of dispersion is that variance is expressed in terms of squared units of returns. This is not understood by most people. The standard deviation overcomes this problem by simply taking the square root of the variance, thus producing a statistic expressed in the same units as the expected value.

The expected return on any portfolio is the weighted average of the expected rates of return for the individual securities in the portfolio. The weights are the proportions (i.e. the percentages) of total funds invested in each security, and the sum of the weights represents the 100% of the total invested capital. Consequently:

$$\text{E}\left(R_p\right) = \overline{R_p} = \sum_{i=1}^{N} w_i E\left(R_i\right) \qquad (6.3)$$

where $E(R_p)$ [or $\overline{R}_p$] = the expected (or average) portfolio return, w_i= the percentage of invested value placed in security i, $E(R_i)$ = the expected return on security i, and N = the number of securities contained in the portfolio.

The risk of a portfolio of assets is a function of the weighted average of the individual variances (where the weights are squared), plus the weighted covariances between all the assets in the portfolio. If iIj, the portfolio risk is:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad (6.4)$$

where σ_p^2 = the variance of the portfolio, σ_i^2 = the variance of rates of return for asset i, the σ_{ij} = the covariance between the rates of return for assets i and j, w_i and w_j = the percentage of investable funds invested in assets i and j, and N = the total number of assets i and j in the portfolio. The double summation sign indicates that the covariance between each pair of assets in the portfolio enters the expression for the variance of a portfolio. Further note that each covariance term is multiplied by two times the product of the proportions invested in each asset.
If i = j, the previous equation can also be written as follows[3]:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad (6.5)$$

Note that **covariance** is an absolute measure of risk and shows how two variables "move together" relative to their individual mean values over time. In portfolio

[3] *If i = j, the covariance between the rates of return for asset i with itself produces the variance of rates of return for this asset.*

analysis we are usually concerned about the covariance of the rates of return of the securities. However, covariance is affected by the variability of the two individual return series. Suppose we examine the rates of return for two securities during a specific time period, and we find that the covariance of their returns is 4.5. This number might indicate a weak positive relationship if the two individual series were very volatile, or it would reflect a strong positive relationship if the two series were very stable. For this reason, it is preferable to “standardize” the covariance, taking into consideration the variability of the two individual return series. This can be done by dividing covariance by the product of the standard deviations of the two securities’ returns. In this case, we get the **correlation coefficient** of the two securities’ returns, which is:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \qquad (6.6)$$

where ϱ_{ij} = the correlation coefficient between the return on security i and the return on security j.

The correlation coefficient can have values ranging from –1 to +1 ($-1 \leq \varrho \leq +1$). A value of $\varrho = -1$ indicates a perfect negative linear relationship between the two return series, which means that the returns tend to move in the opposite direction (when the one increases, the other decreases, and when the one decreases, the other increases)[4]. Consequently, if we know that the return of a security is high, we can predict that the return of the other security will be low. A value of $\varrho = +1$ indicates a perfect positive linear relationship between the two return series, meaning that the returns of the two securities move together in a completely linear manner (when the one increases, the other will also increase, and when the one decreases, the other will also decrease)[5]. Therefore, when the return of a security is high, the return of the other security will also be high. If $\varrho = 0$, there is no linear relationship between the returns of the two securities. Whenever the returns are uncorrelated statistically, knowledge of the return on one security is of no value in predicting the return of the second security. This, of course, does not mean that the security returns are independent[6].

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[4] *A negative correlation is postulated by economic theory for the quantity of a commodity demanded and its price. When price increases, demand for the commodity decreases and when price falls demand increases.*

[5] *A positive correlation is postulated by economic theory for the quantity of a commodity supplied and its price. When price increases the quantity supplied increases, and conversely, when price falls the quantity supplied decreases.*

[6] *Whenever two random variables are independent, their covariance and their correlation coefficient are zero. The opposite, however, does not essentially hold. This is because covariance measures linear correlation, whereas the relationship between the variables can be of a different type.*

According to the above, the portfolio's risk equation (if i≠j) can be formulated as follows:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j \qquad (6.7)$$

The simplest case is when the examined portfolio consists of two assets (1 and 2). In this case the portfolio risk equals to:

$$\sigma_p = \left[w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12} \sigma_1 \sigma_2 \right]^{1/2} \qquad (6.8)$$

Therefore, when estimating portfolio risk, we should consider the following:

- the weighted variances of the assets in the portfolio (i.e. the risk of each individual asset), and
- the weighted covariances between all the assets in the portfolio.

One of Markowitz's real contributions to portfolio theory is that he proved the relative importance of the two factors mentioned above. As the number of assets held in the portfolio increases, the importance of each individual asset's risk (i.e. its standard deviation) decreases, whereas the importance of the covariance among the assets' returns increases. Consequently, when we add a security in a portfolio which already contains a number of other securities, the average covariance of this new security's return with the returns of all the other securities is more important (i.e. it has a greater impact on the portfolio risk) than the individual security risk (i.e. its variance), and the more assets in the portfolio, the more this is true. The more securities a portfolio contains, the higher the relative importance of the average covariance of the added security with the returns of all the other securities contained in the portfolio.

The above analysis shows the importance of asset correlations. When an investor simply invests in a number of different assets and hopes that the variance of the expected return on the portfolio is lowered (e.g. by purchasing stocks of several different industries), then this strategy is called **naïve diversification**. On the other hand, **Markowitz diversification** seeks to combine assets in a portfolio with returns that are less than perfectly correlated, in an effort to lower portfolio risk without sacrificing return. As the correlation between the returns for assets that are combined in a portfolio decreases, so does the variance of the return for that portfolio. An investor can maintain expected portfolio return and can lower portfolio risk by combining assets with lower correlations. However, there are very few assets that have small to negative correlations with other assets. In the U.S.A., the correlation coefficient of the majority of stocks ranges from 0.5 to 0.6.

6.2 DETERMINATION OF EFFICIENT PORTFOLIOS

The Markowitz model is based on several assumptions about asset selection behavior, which are the following:

1. There are only two parameters that affect an investor's decision, the expected return and the variance.
2. Investors are risk averse.
3. All investors seek to achieve the highest expected return at a given level of risk.
4. All investors have a common one-period investment horizon.
5. All investors have the same expectations regarding expected return, variance, and covariances for all risky assets.

Under these assumptions Markowitz was able to construct efficient portfolios. To see how let us form an infinite number of possible portfolios from a set of many assets. If we plot these possible portfolios in the risk-return posture (i.e. the vertical axis is expected return and the horizontal axis is standard deviation), will have an umbrella-type shape similar to the one shown in figure 1. This area of all possible portfolios is called **attainable set (or opportunity set or feasible set)** of portfolios. Consequently, the attainable set of portfolios is the collection of all portfolios that can be formed from the group of securities being considered by an investor.

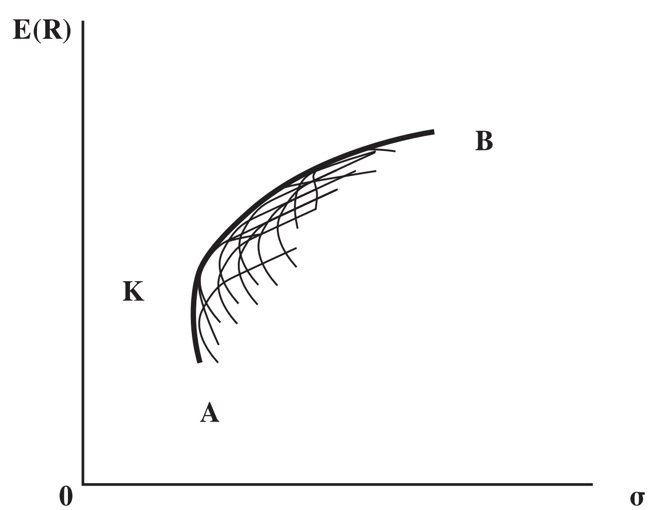

Figure 1: Attainable set of portfolios

However, there is a set of portfolios that dominates all other feasible portfolios because they offer the maximum rate of return for every given level of risk, or the minimum risk for every level of return. These portfolios are called **efficient portfolios** (or mean-variance efficient portfolios). Thus, an efficient portfolio is a portfolio that provides the greatest expected return for a given level of risk or, equivalently, the lowest risk for a given expected return. The collection of all efficient portfolios is called efficient set of portfolios and consists of the envelope curve of all portfolios that lie between the minimum variance portfolio and the maximum return portfolio. In figure 1 all the efficient portfolios lie on the northwest boundary between points K and B, and K is the minimum variance portfolio. The efficient set of portfolios is sometimes called the **efficient frontier**, because graphically all the efficient portfolios lie on the boundary of the set of feasible portfolios that have the maximum return for a given level of risk. In order to construct the efficient frontier a mathematical technique called quadratic programming must be used. In figure 1 the curve KB represents the efficient frontier. Thus, the efficient frontier represents that set of portfolios that has the maximum rate of return for every given level of risk, or the minimum risk for every level of return.

6.3 SELECTING THE OPTIMAL PORTFOLIO

The Markowitz model determines the efficient frontier which represents a set of the efficient portfolios. However, which is the specific efficient portfolio that an investor prefers? The best portfolio to hold of all those on the efficient frontier is called **optimal portfolio** and depends on the investor's preferences, concerning the trade–off between return and risk. These preferences are represented by the **utility function** of each investor. A utility function can be expressed in graphical form by a curve. This curve specifies the trade-off between return and risk that provide an investor with the same amount of utility, and is called **indifference curve**. The optimal portfolio is the efficient portfolio that has the highest utility for a particular investor, and lies at the point of tangency between the efficient frontier and the curve with the highest possible utility. The question, however, is how we can estimate the utility function of an investor, in order to determine his/her optimal portfolio. Unfortunately, economists have not been successful in measuring utility functions.

Figure 2 shows two optimal portfolios, X and Y, for two hypothetical investors. Portfolio X is optimal for a very risk averse investor (i.e. a conservative investor), who is not willing to tolerate much additional risk to obtain higher returns. On the contrary, portfolio Y is optimal for a less risk averse investor (i.e. an aggressive investor), who is willing to tolerate much more risk in order to obtain higher return. In general, conservative investors tend to select portfolios which are closer to the left point of the efficient frontier KB. These portfolios have both lower risk and lower return. Risky investors tend to select portfolios which are closer to the right point of the efficient frontier KB. These portfolios provide higher returns and, consequently, higher risk.

The major problem with the Markowitz model is that many calculations are needed. When a portfolio consists of (n) securities, we have to estimate (n) expected returns, (n) variances and [n(n-1)]/2 covariances, which sum at [n(n+3)]/2 calculations. Hence, for a portfolio of 100 securities 5,150 parameters would need to be estimated. For this reason, the Markowitz model remained primarily of academic interest, until the estimations of its inputs were simplified by the **single-index model**.

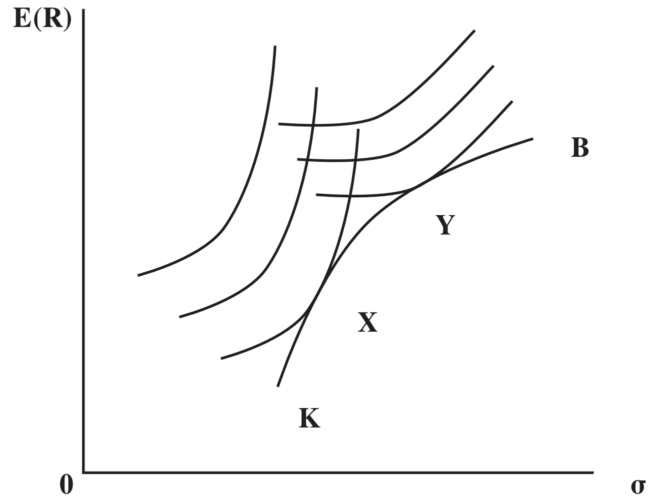

Figure 2: Optimal portfolio selection

6.4 THE SINGLE-INDEX MODEL

The single-index model was developed by William Sharpe[7] and considerably diminishes the number of calculations needed for the estimation of the efficient frontier. This model assumes that security returns are correlated because they are influenced by the general economic conditions and not because of their particular characteristics. Consequently, the model assumes that all stocks (and generally securities) have a common response to market changes. As a result, the return on a stock should be related to the return on a market index which reflects market changes. Although this index can be any variable, in this model we usually use a stock market index. The single-index model can be stated as

$$R_i = \alpha_i + \beta_i R_m + e_i \qquad (6.9)$$

where R_i = the rate of return on security i, R_m = the rate of return on the market index, α_i = a component of security i's return that is independent of the market's performance (that is, a random variable), β_i = a coefficient which measures the sensitivity of the security i's return to a change on the return on the market index, and e_i = a random error term (or the difference between the actual security's return and the expected return given the market return)[8].

The single-index model can be estimated by a **simple linear regression** of the security i's return to the return of the index. Graphically, the single-index model can be depicted as a line fitted to a plot of security returns against returns on the market index. This is shown for a hypothetical security in figure 3.

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[7] *For more information see Sharpe (1963).*
[8] *From econometrics it is known that the error term or random disturbance term or stochastic term shows the deviations of the real observations from the line. Residuals are estimations of the error term and show the deviations of the dependent variable observations from the estimated value. In other words, error terms are related with the real regression model, whereas residuals come from the estimation procedure.*

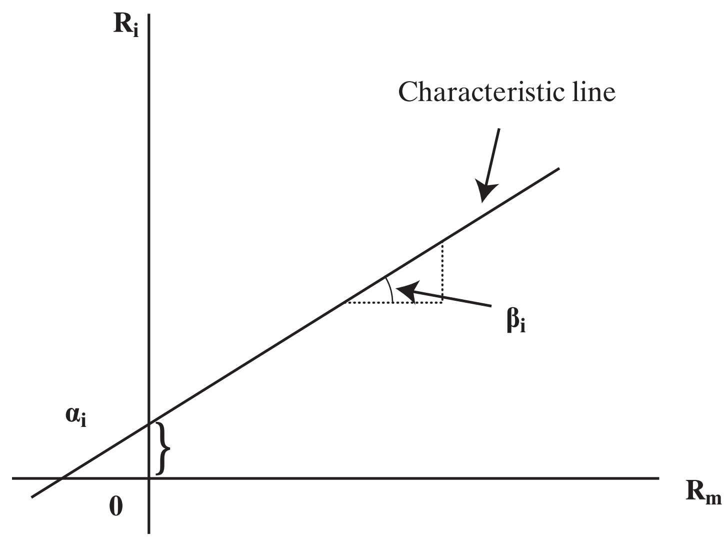

Figure 3: Characteristic line

The straight regression line of the single-index model is called **characteristic line** and represents the relationship between the changes of a security's returns (i.e. a stock) and the changes of a market index's returns. The slope of this line is called **beta coefficient** and is the regression coefficient. It measures the degree to which the returns on the security change systematically with changes in the market index's returns. Hence, the beta coefficient is referred to as an index of that systematic risk due to general market conditions that cannot be diversified away. The beta for the market portfolio is 1.0. The term α, popularly referred to as alpha, is the intercept point on the vertical axis. It is equal to the average value over time of the unsystematic returns for the security. The beta coefficient and the constant term (i.e. the alpha) are estimated using the ordinary least squares (OLS) as follows:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \qquad (6.10)$$

$$\alpha_i = E(R_i) - \beta_i E(R_m) \qquad (6.11)$$

Furthermore, the correlation coefficient between the security and the index is:

$$\varrho_{im} = \frac{\sigma_{im}}{\sigma_i \sigma_m} \Rightarrow \varrho_{im} = \frac{\beta_i \sigma_m^2}{\sigma_i \sigma_m} \Rightarrow \varrho_{im} = \beta_i \frac{\sigma_m}{\sigma_i} \qquad (6.12)$$

In case we replace i with m, in equation (6.10) [or equation (6.12)], we see that the beta of the market index is one.

$$\beta_m = \frac{\sigma_{mm}}{\sigma_m^2} = \frac{\sigma_m^2}{\sigma_m^2} = 1 \qquad (6.13)$$

If the beta coefficient of a security is higher than one, the security is considered as aggressive, since a 1% change in the return of the market index will result in even higher changes in the return of the security. On the contrary, if the beta coefficient of a security is lower than one, the security is considered as defensive, because its return is less influenced by changes in the return of the market index. Note that the beta coefficient is measured in a ratio scale. In other words, a beta coefficient of 2 implies that it is twice as much as a beta coefficient of 1, whereas a beta coefficient of 0.5, is half as much as a beta coefficient of 1.

6.5 USE OF THE SINGLE-INDEX MODEL

The single-index model can be used in two different ways:

(a) To simplify the estimations needed in the Markowitz model. By using the single-index model, we can easily estimate the expected return on each security, the standard deviation on each security, and the covariances between each pair of securities, in order to generate the efficient frontier of portfolios. In this case, the following equations provide the necessary inputs to the Markowitz model[9]:

$$E(R_i)=\alpha_i+\beta_i E(R_m) \qquad (6.14)$$

$$\sigma_i^2=\beta_i^2\sigma_m^2\sigma_{ei}^2 \qquad (6.15)$$

$$\sigma_{ij}=\beta_i\beta_j\sigma_m^2 \qquad (6.16)$$

where σ_{ei}^2= the residual risk (variance) of the securities. To implement the above equations, we need the estimations of α_i, β_i and σ_{ei}^2 for every security, as well as the estimations of $E(R_m)$ and σ_m^2. As a result, if we have (n) securities, we need (3n+2) calculations. For example, for a portfolio of 100 securities, we have to make 302 calculations.

(b) To solve the problem of portfolio analysis directly, by calculating the expected return and risk for portfolios[10]. Rather than estimate the inputs for the Markowitz model, we can directly estimate the expected return and risk for a portfolio based on relationships involved with the single-index model. The expected return on any portfolio is estimated as follows:

$$E(R_p)=\alpha_p+\beta_p E(R_m) \qquad (6.17)$$

where $\alpha_p=\sum_{i=1}^{n} w_i\alpha_i$ and $\beta_p=\sum_{i=1}^{n} w_i\beta_i$

Parameters α_p and β_p of the portfolio are the weighted averages of each security's α and β, where the weights are the fraction of the portfolio invested in each security. The portfolio risk is estimated as follows:

$$\sigma_p^2=\beta_p^2\sigma_m^2+\sum_{i=1}^{n} w_i^2\sigma_{ei}^2 \qquad (6.18)$$

where $\beta_p=\sum_{i=1}^{n} w_i\beta_i$

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[9] *For the derivation of the following equations see Elton, Gruber, Brown and Goetzmann (2003), pp. 134-137.*

[10] *For the derivation of the following equations see Elton, Gruber, Brown and Goetzmann (2003), pp. 134-137.*

As the number of securities in the portfolio increases, the importance of the residual risk diminishes drastically, whereas the importance of the market risk increases. Take for example the second term in the right–hand side of the above equation and suppose that the amounts invested in all the securities are equal, i.e. $w_i = (1/N)$. In this case the second term can be written as follows:

$$\sum_{i=1}^{N} w_i^2 \sigma_{ei}^2 = \sum_{i=1}^{N} \left(\frac{1}{N}\right)^2 \sigma_{ei}^2 = \frac{1}{N}\left[\sum_{i=1}^{N} \frac{\sigma_{ei}^2}{N}\right] = \frac{1}{N}\left[\overline{\sigma_{ei}^2}\right]$$

The term $[\overline{\sigma_{ei}^2}]$ shows the average residual variance. As the number of securities in the portfolio increases, the average unique risk of the component securities $\{(1/N)[\overline{\sigma_{ei}^2}]\}$ declines drastically and approaches to zero (in case N is large). Consequently, the risk of a portfolio which consists of many securities is derived mainly from the first term of the above equation. In this case, the risk of the portfolio approaches

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 \qquad or \qquad \sigma_p = \beta_p \sigma_m = \left[\sum_{i=1}^{n} w_i \beta_i\right] \sigma_m \qquad (6.19)$$

Since σ_m is the same for all securities (or portfolios), β_i measures the contribution of each security to the risk of a large portfolio.

From the above it follows that the more diversified a portfolio (that is, the larger the number of securities in the portfolio), the smaller will be its unique risk (that is, the residual risk) and, in turn, its total risk. The risk that is not eliminated as we hold larger and larger portfolios is the risk associated with the term β_p. The term β_p will neither decrease nor increase significantly by increasing the amount of diversification. Thus, diversification can substantially reduce unique risk, while leads to an averaging of market risk.

6.6 SYSTEMATIC AND UNSYSTEMATIC RISK

In the previous section we saw that the risk of an individual security could be partitioned into two components as follows:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

We also saw that the effect of residual risk (σ_{ei}^2) on portfolio risk approaches zero as the portfolio gets larger. This is the reason why this risk is called **unsystematic risk.** It is also sometimes called diversifiable risk, unique risk, residual risk, company-specific risk or non-market risk. The effect, however, of the first term ($\beta_i^2\sigma_m^2$) on the portfolio risk, does not diminish as the number of securities in the portfolio gets larger. Since σ_m (i.e. the standard deviation of the market portfolio) is constant with respect to all securities, β_i is the measure of a security's **systematic risk**. It is also sometimes called non-diversifiable risk or market risk. Systematic risk is the variability of returns that is due to macroeconomic factors that affect all risky assets. Examples of such macroeconomic factors are the variability of growth in the money supply, the interest rate volatility, business cycles, the industrial production variability and the earnings variability of the firms. Because the systematic risk affects all risky assets, it cannot be eliminated by diversification. Since the unsystematic risk can be eliminated by holding diversified portfolios (i.e. portfolios consisting of a large number of assets), β_i is often used as the measure of a security's risk. From the above it follows that

$$\sigma_i^2 = \text{Systematic risk} + \text{Unsystematic risk}$$

Consequently, the total risk of a security consists of two parts, the systematic or non-diversifiable risk and the unsystematic or diversifiable risk. The unsystematic risk can be eliminated – or, at least, reduced substantially – by holding a portfolio with several stocks. In effect, the unique part of the risk of each security is canceled out or smoothed, leaving the part that is attributable to the systematic variance arising from the overall market. Hence, diversification is the process of adding securities to a portfolio in order to reduce the portfolio's unique risk and, thereby, the portfolio's total risk, without sacrificing return. However, the question is how does one do this in practice? Recall that there is naïve diversification and Markowitz diversification. The Markowitz diversification strategy would advice an investor to combine assets in a portfolio with returns that are less than perfectly correlated, in an effort to lower portfolio risk without sacrificing return. Nevertheless, a well-diversified portfolio will have only systematic risk, since the unsystematic risk will be eliminated. But how many securities does it take to eliminate the unsystematic risk? Evans and Archer (1968) estimated the standard deviation for portfolios that have equal proportions [w=(1/N)] of increasing number of stocks randomly selected. According to this study, the total risk of a 12-18 stock portfolio was approximately the same as that for the market portfolio. Subsequent studies conclude that a well diversified stock portfolio must include at least 20 to 30 stocks[11]. As far as the Greek Stock Market (Athens Exchange) is concerned, Papaioannou and Milonas (1983) found that about 10 stocks are enough to eliminate the unsystematic portfolio risk. Figure 4 shows a graph of the effect.

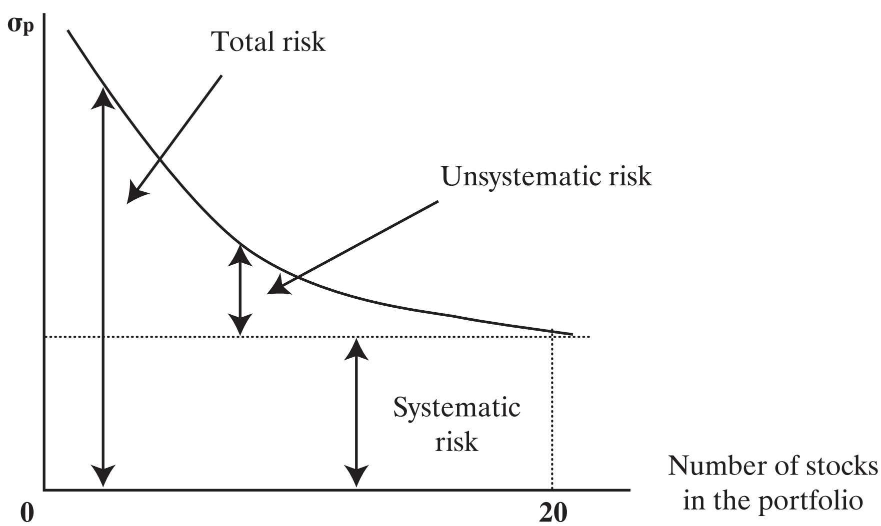

Figure 4: Systematic and unsystematic portfolio risk

Activity 1/Chapter 6

Assume that the expected return on the market is 12% with a standard deviation of 20%. The following information is available for stocks K and H:

Stocks	α_i	β_i	Residual Error or σ_{ei}^2
K	15%	1.2	720
H	4%	0.8	320

Using the single-index model, answer the following:

a) Calculate the expected return for each stock.

b) Calculate the variance, and standard deviation of returns for each stock, as well as the covariance and the correlation coefficient between the rates of return.

[11] *See Wagner and Lau (1971), and Statman (1987).*

c) Which of the two stocks is the riskier if 100% of an investor's wealth can be invested in only one stock?

d) Which stock is least risky when added to a well diversified portfolio?

e) Calculate the expected return and the standard deviation of a portfolio consisting of 30% of stock K and 70% of stock H.

Synopsis

- **Portfolio theory is concerned with an investor's portfolio, i.e. the combination of assets invested in and held by an investor. The basic portfolio theory was developed by Harry Markowitz and is based on specific characteristics of securities.**
- **The expected return on any portfolio is the weighted average of the expected rates of return for the individual securities in the portfolio. The weights are the proportions (i.e. the percentages) of total funds invested in each security, and the sum of the weights represents the 100% of the total invested capital.**
- **The risk of the portfolio is determined by three factors: a) the risk of each individual security (i.e. the individual variances), b) the covariances between all the securities in the portfolio, and c) the weights (percentage of investable funds) given to each security. As the number of securities held in the portfolio increases, the importance of each individual security's risk (i.e. its standard deviation) decreases, whereas the importance of the covariances between all the securities in the portfolio increases.**
- **The attainable set of portfolios is the collection of all portfolios that can be formed from the group of securities being considered by an investor. An efficient portfolio is a portfolio that provides the greatest expected return for a given level of risk or, equivalently, the lowest risk for a given expected return. The collection of all efficient portfolios is called efficient set of portfolios and consists of the envelope curve of all portfolios that lie between the minimum variance portfolio and the maximum return portfolio. The efficient set of portfolios is sometimes called the efficient frontier, because graphically all the efficient portfolios lie on the boundary of the set of feasible portfolios that have the maximum return for a given level of risk.**
- **The optimal portfolio is the efficient portfolio that has the highest utility for a particular investor, and lies at the point of tangency between the efficient frontier and the curve with the highest possible utility.**
- **This single-index model assumes that security returns are correlated because they are influenced by the general economic conditions and not because of their particular characteristics. Consequently, the model assumes that all stocks (and generally securities) have a common response to market changes. As a result, the return on a stock should be related to the return on a market index which reflects market changes. Although this index can be any variable, in this model we usually use a stock market index (as for example, the S&P 500 index).**
- **The single-index model can be estimated by a simple linear regression of the security i's return to the return of the index. Graphically, the single-index model can be depicted as a line fitted to a plot of security returns against returns on the market index. The straight regression line of the single-index model is called characteristic line and represents the relationship between the changes of a security's returns (i.e. a stock) and the changes of a market index's returns. The slope of this line is called beta coefficient and is the regression coefficient. It measures the degree to which the returns on the**

security change systematically with changes in the market index's returns. Hence, the beta coefficient is referred to as an index of that systematic risk due to general market conditions that cannot be diversified away.

- **The single-index model can be used in two different ways: (a) To simplify the estimations needed in the Markowitz model. (b) To solve the problem of portfolio analysis directly, by calculating the expected return and risk for portfolios.**
- **Diversification is the process of adding securities to a portfolio in order to reduce the portfolio's unique risk and, thereby, the portfolio's total risk, without sacrificing return.**
- **The total risk of a security consists of two parts, the systematic or non-diversifiable risk and the unsystematic or diversifiable risk. The unsystematic risk can be eliminated – or, at least, reduced substantially – by holding a portfolio with several stocks. In effect, the unique part of the risk of each security is canceled out or smoothed, leaving the part that is attributable to the systematic variance arising from the overall market.**
- **At a portfolio size of about 20 randomly selected common stocks, the level of total portfolio risk is reduced such that all is left is systematic risk.**

Answer to the Activity

Activity 1

Answer:

a) $E(R_i) = a_i + \beta_i E(R_m) \Rightarrow$

$E(R_K) = 15 + (1.2)\times(12) = 29.4\%$

$E(R_H) = 4 + (0.8)\times(12) = 13.6\%$

b) $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2 \Rightarrow$

$\sigma_K^2 = (1.2)^2\times(20)^2 + 720 = 1{,}296 \Rightarrow \sigma_K = 36\%$

$\sigma_H^2 = (0.8)^2\times(20)^2 + 320 = 576 \Rightarrow \sigma_H = 24\%$

$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \Rightarrow \sigma_{KH} = (1.2)\times(0.8)\times(20)^2 = 384$

$\rho_{ij} = (\sigma_{ij} / \sigma_i\sigma_j) \Rightarrow \rho_{KH} = [384/(36)\times(24)] = 0.4444$

c) If 100% of an investor's wealth can be invested in only one stock, the K stock is the riskier, since σ_K (= 36%) > σ_H (= 24%).

d) If a stock is added to a well-diversified portfolio, the risk of the portfolio increases by the systematic risk of the stock. Consequently, to answer this question, we must calculate the systematic risk of each stock.

Systematic risk = $\sigma_{systematic,i}^2 = \beta_i^2 \sigma_m^2 \Rightarrow \sigma_{systematic,i} = \beta_i \sigma_m \Rightarrow$

$\sigma_{systematic,K} = (1.2) \times(20) = 24\%.$

$\sigma_{systematic,H} = (0.8) \times(20) = 16\%.$

Stock H has lower systematic risk and, therefore, adds less risk to a well-diversified portfolio. Thus, stock H should be added to a well-diversified portfolio.

e) $\alpha_p = \sum w_i \alpha_i = [(0.30)\times(15)] + [(0.70)\times(4)] = 7.3$

$\beta_p = \sum w_i b_i = [(0.30)\times(1.2)] + [(0.70)\times(0.8)] = 0.92$

The expected portfolio return is:

$E(R_p) = \alpha_p + \beta_p E(R_m) \Rightarrow E(R_p) = (7.3) + [(0.92)\times(12)] = 18.34\%$

The portfolio risk is:

$\sigma_p^2 = \beta_p^2\sigma_m^2 + \sum w_i^2\sigma_{ei}^2 \Rightarrow \sigma_p^2 = [(0.92)^2\times(20)^2]+[(0.30)^2\times(720)]+[(0.70)^2\times(320)] \Rightarrow$

$\sigma_p^2 = 560.16 \Rightarrow \sigma_p = 23.67\%.$

The portfolio risk can also be estimated from the Markowitz model, as follows:

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad or \qquad \sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \rho_{ij}\sigma_i\sigma_j \Rightarrow$$

$$\sigma_p^2 = w_K^2\sigma_K^2 + w_H^2\sigma_H^2 + 2w_K w_H \rho_{KH}\sigma_K\sigma_H \Rightarrow$$

$$\sigma_p^2 = [(0.30)^2 \times (36)^2] + [(0.70)^2 \times (24)^2] + [(2) \times (0.30) \times (0.70) \times (0.4444) \times (36) \times (24)]$$

$$\sigma_p^2 = 560.14 \Rightarrow \sigma_p = 23.67\%$$

(Note: The differences are ought to the fact that the figures have been rounded off to the nearest decimal).

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CHAPTER 7

CAPITAL MARKET THEORY

The Scope of the Chapter

In this chapter, we introduce the capital market theory. We show how various assets are evaluated. This valuation is conducted with the help of some specific models.

Learning Objectives

When you have finished studying this chapter, you will be able to:
- mention the basic assumptions of the Capital Asset Pricing Model – CAPM
- define the risk-free asset
- estimate the expected return and risk when combining the risk-free asset with a risky asset portfolio
- define the security market line – SML and how it differs from the capital market line
- define the market portfolio and the assets it contains
- develop the separation theorem
- dontrast overvalued and undervalued securities (using the SML) and explain how a security is determined as overvalued or undervalued
- mention the most important conclusions of the CAPM's empirical studies.

Key Words

- Capital market theory
- Risk-free asset
- Capital market line
- Market price of risk
- Market risk premium
- Security market line - SML
- Capital asset pricing model
- Market portfolio
- Separation theorem
- Characteristic line in risk premium (or excess form)

Introductory Comments

The capital market theory can be considered as an extension of Markowitz portfolio theory. Portfolio theory describes how investors can create effective portfolios that include various assets. Capital market theory examines how assets should be valued in the capital market, provided that investors behave according to the assumptions of portfolio theory trying to maximize their utility. Capital market theory includes various models; the most important of these models is the Capital Asset Pricing Model. This model assumes that there is a relationship between the expected rate of return of an asset and its systematic risk. Capital market theory is very important in investment analysis and portfolio management.

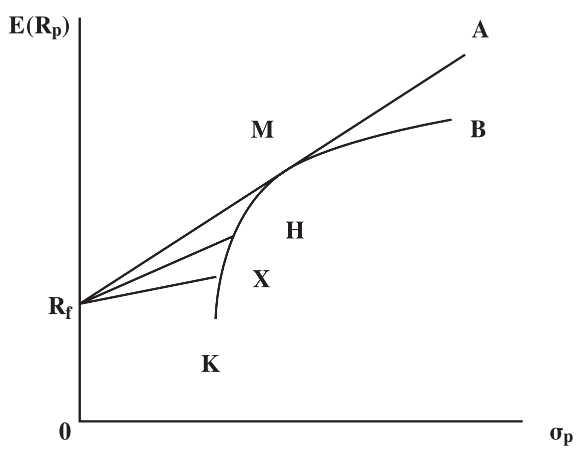
This chapter includes six sections. The first section analyzes how the efficient frontier is determined, when we introduce in our analysis the risk-free asset. The second section examines the market portfolio. The third section describes the separation theorem and the fourth refers to the security line. The fifth section describes how the Capital Asset Pricing Model is used. Finally, the sixth section describes the most important empirical studies about the Capital Asset Pricing Model.

7.1 THE CAPITAL MARKET LINE

Capital market theory uses the Markowitz portfolio theory to evaluate various assets. Therefore, capital market theory is based on portfolio theory.

Suppose that an investor used portfolio theory and created an efficient frontier. Also suppose that there is a risk-free asset, which provides return R_f with zero risk ($\sigma_{Rf} = 0$). The investor can combine an investment in the risk-free asset with an investment in any portfolio X that lies on the efficient frontier of Markowitz. Suppose that he/she invests a part of his portfolio in the risk-free asset and the remainder in portfolio X. In this case, the more the capital invested in X, the higher the expected return and the risk. Therefore, the graphic depiction of the combinations of expected return and risk of these two assets (i.e. the risk-free asset and portfolio X) will be a straight line connecting these two assets. Diagram 1 shows the various combinations of return and risk for combinations of the risk-free asset with portfolios that lie on the efficient frontier. If this portfolio is the X, then these combinations lie on the straight line R_fX. The investor can choose any combination that lies on R_fX, by investing a part of his/her portfolio in the risk-free asset and the remainder in portfolio X. All combinations that lie on R_fX dominate (i.e. they are more efficient than) the combinations of the risk-free asset with portfolios that lie on the efficient frontier but below portfolio X. This happens because for each portfolio that lies on the efficient frontier but below portfolio X, there is another on R_fX (i.e. a combination of the risk-free asset and the portfolio X) with the same standard deviation but higher expected return. Similarly, the investor can combine the risk-free asset with another portfolio, say H, which lies in the efficient frontier but at a higher point than the X. The straight line R_fH shows all possible combinations of these two assets.

Diagram 1
Combinations of return and risk when a risk-free asset is incorporated in the analysis



Each point of line R_fH dominates (i.e. it is more efficient than) the corresponding points of the previous R_fX line. The same process continues until the straight line is tangent to KB, (which is the efficient frontier); this occurs in point M. The straight line that connects R_f with M, dominates everything below the line in the original efficient frontier. Furthermore, there can be no lines about R_fM. Line R_fM includes all the efficient portfolios where a part of them is invested in portfolio M and the rest in the risk-free asset.

However, investors may also borrow at the risk-free rate and invest these funds in the risky asset portfolio (i.e. portfolio M). In this case, the efficient frontier has the form of line R_fMA. These investors undertake more risk than those that invested part of their capital to the risk-free asset (i.e. they lent their funds at the R_f) and consequently their expected return is higher than the portfolio's M return. The efficient frontier is the straight line R_fMA because both, the expected return and the risk of the portfolio, increase linearly. The straight line R_fMA, which is tangent to previous efficient frontier KB in point M, is the **capital market line** – CML and shows the trade-off between the expected return and the risk for all the efficient portfolios. The capital market line is depicted in diagram 1.
The capital market line has the following form:

$$E(R_p) = R_f + \frac{[E(R_m) - R_f]}{\sigma_m}\sigma_p \qquad (7.1)$$

where, $E(R_p)$ = the expected return of the efficient portfolio p, R_f = the return of the risk-free asset, $E(R_m)$ = the expected return of market portfolio m, σ_m = the standard deviation of market portfolio, and σ_p = the standard deviation of portfolio p.

The slope of the capital market line is $[E(R_m) - R_f] / \sigma_m$ and is known as **market price of risk** of the efficient portfolios. The numerator of this ratio is the expected return of the market portfolio minus the return of the risk-free asset. This is a compensation to the holder of the market portfolio for his/her undertaking risk and is known as **market risk premium**. The denominator is the risk of the market portfolio. Thus, the slope of the capital market line measures the compensation per unit of risk for the market portfolio. Because the capital market line represents the return offered as compensation for each level of risk, each point in the capital market line is the market's equilibrium point. The slope of the line determines the required additional return in order to compensate the investor for each change in risk that he/she undertakes. Thus, this slope shows the required additional return from the market for each percentage increase in the risk of the efficient portfolio.

7.2 THE MARKET PORTFOLIO

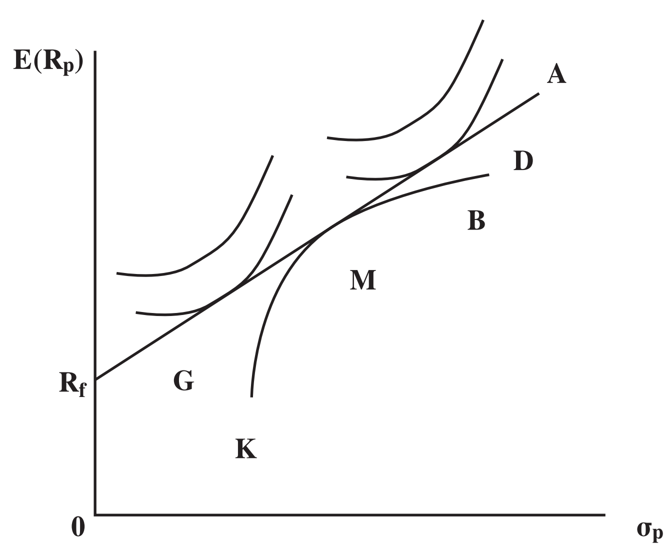
Portfolio M is known as the **market portfolio** and lies at the point of tangency of the capital market line and the risky assets efficient frontier. Portfolios that lie on the risky assets efficient frontier (KB) are less effective than portfolio M. Thus, all investors will invest in portfolio M and in the risk-free asset and, consequently, their portfolios will lie on the efficient frontier R_fMA. The portfolio of each investor in line R_fMA (and consequently the return and risk of each investor) is determined by the portion of his portfolio invested in the risk-free asset and in portfolio M.

Because the market is in equilibrium, all risky assets are included in portfolio M in proportion to their market value. In other words, the market portfolio is a portfolio in which each asset is included at this specific percentage determined by the market value of the asset divided by the total market value of all risky assets. If, for example, the value of McDonald's in the market is 2% of the total value of all risky assets, then the 2% of the market portfolio M will be invested in shares of McDonald's. Actually, the market portfolio cannot be determined, as it includes shares, bonds, gold, currencies, real estate, antiques, stamps etc. Because the market portfolio includes all risky assets, it is completely diversified and consequently has only systematic risk. A simplified version of the market portfolio is the one that includes all outstanding shares of all firms; however, the concept of the market portfolio is even more simplified, by using specific indices as proxies, like the S&P 500 or the FTSE 20.

7.3 THE SEPARATION THEOREM

The separation theorem was initially proposed by James Tobin (1958) and it refers to the division of the investor's investment decision from the financing decision. More specifically, each investor takes an investment decision, which means that he/she decides to invest in the market portfolio (M). Then, each investor takes a decision of financing his/her investment in portfolio M and based on his/her risk preferences (i.e. the distribution of his/her capital in the risk-free asset and the market portfolio) determines the point on the efficient frontier R_fMA that he/she is willing to invest. In other words, the separation theorem says that the determination of the market portfolio is independent from the investors' preferences on risk and return. The optimal investment policy for each investor is determined by the point in which the highest indifference curve is tangent to the capital market line. For example, an investor, who is risk averse, selects portfolio G, while another investor who prefers more risk would select the portfolio D, as diagram 2 shows.

Diagram 2
Choice of the optimal portfolio

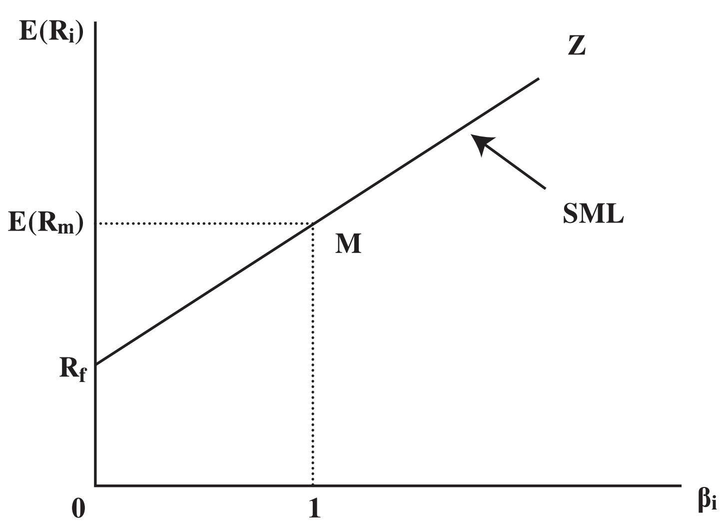


7.4 THE SECURITY MARKET LINE

The previous analysis showed how an investor, who is risk averse and takes decisions based on his/her expected return and risk, can form an efficient portfolio combining the market portfolio and the risk-free asset. This result helps us create a model that shows how an asset can be evaluated.

Portfolio theory says that if a security is added in a well diversified portfolio, the risk added to the portfolio risk is only the systematic risk of the security which is equal to [$\beta_i^2 \sigma_m^2$]. This portfolio risk is proportional to the beta coefficient (β_i) of the security[1], which is the slope of the characteristic line of the security. Bearing in mind that the contribution of each security to the total risk of a well diversified portfolio is the systematic risk of the security (that is related directly to the beta coefficient of the security), we can graphically depict the trade-off between the expected return and the risk of each security using the beta coefficient, instead of the standard deviation in the horizontal axis. This is presented in diagram 3.

Diagram 3
Security market line



At a first glance we have the impression that the vertical axis of the diagram 3 depicts the expected return of each security. However, this is only half the reality. In equilibrium, to buy a security investors require to get a minimum expected

[1] *Beta coefficient measures the relative systematic risk.*

return. Furthermore, it is known that the required return is the minimal expected return that a security should have so that investors would buy it. Therefore, each security should have a minimum expected return, given its risk, in order to attract investors. Thus, the vertical axis of diagram 3 depicts expected and required returns simultaneously. Line R_fMZ is the **security market line**-SML and shows the trade-off between the expected (and the required return) and the systematic risk, for all securities (either they only have systematic risk or not) and all portfolios (they are either effective or not).
The security market line has the following form:

$$E(R_i)=R_f+[E(R_m)-R_f]\beta_i \qquad (7.2)$$

Equation (7.2) is also reported as the algebraic function of the **capital asset pricing model** – CAPM, developed by William Sharpe, John Lintner and Jan Mossin[2]. Generally, the capital asset pricing model states that an investor requires the expected return of a risky asset to be equal to the return of a risk-free asset plus an extra return for the systematic risk that he undertakes with the purchase of the particular asset. The higher the systematic risk of the asset, the higher this extra return.

Example 1

We have the following information about five mutual funds, which we suppose are efficient.

Mutual funds	Standard deviations (%)
A	10
B	14
C	18
D	22
E	26

Furthermore, the expected return of the market portfolio is 12%, the standard deviation is 20 % and the expected return of the risk-free asset is 6%:
(a) Calculate the slope of the capital market line (CML).
(b) Calculate the expected return of each portfolio.
(c) Is there a mutual fund with expected return equal to that of the market?
Answer:
(a) The slope of the capital market line is equal to

$$[E(R_m) - R_f] / \sigma_m = (12 - 6)/20 = 0.30$$

[2] *See Sharpe (1964), Lintner (1965), and Mossin (1966).*

0.30 is the price of the risk of the efficient portfolios in the market. Therefore, 0.30 is the additional return for each percentage increase in the risk of an efficient portfolio.

(b) The expected return of each mutual fund is: $E(R_p)=R_f+\{[E(R_m)-R_f]/\sigma_m\}\times\sigma_p$. Therefore

Mutual funds	Expected returns (%)
A	$E(R_A) = 6+[(12-6)/20]\times(10) = 9$
B	$E(R_B) = 6+[(12-6)/20]\times(14) = 10.2$
C	$E(R_C) = 6+[(12-6)/20]\times(18) = 11.4$
D	$E(R_D) = 6+[(12-6)/20]\times(22) = 12.6$
E	$E(R_E) = 6+[(12-6)/20]\times(26) = 13.8$

(c) There is no mutual fund with expected return equal to that of the market, as well as there is no mutual fund that would have the same risk with that of the market.

Activity 1/Chapter 7

Suppose that investors use the capital asset pricing model (CAPM) and can lend or borrow any amount. The expected return of the market portfolio is 0.12 and the standard deviation is 0.20. The return on the risk-free asset is 0.06.

(a) Fill in the gaps in the following table.

Shares	Expected Return	Standard deviations	Beta coefficient	Residual volatility
A	0.16	–	–	0.0650
B	0.08	–	–	0.0755

(b) Find the expected return, the beta coefficient and the standard deviation of the portfolio if the 4/5 of the funds are invested in share A and the remaining 1/5 in share B.

7.5 USING THE CAPITAL ASSET PRICING MODEL (CAPM)

In order to estimate the expected return of a security (or a portfolio) according to the Capital Asset Pricing Model, we need to estimate the expected return on the risk-free asset (R_f), the market portfolio's expected return $E(R_m)$ and the security's beta coefficient (β_i) (or portfolio's beta coefficient).

In the past, most analysts recommended the short-term Treasury bills to estimate R_f. After 1988 most analysts recommend the use of the medium-term treasury bonds (instead of the Treasury bills) as the risk-free asset. This replacement is based on the opinion that most investors have an investment time horizon longer than that of Treasury bills. More specifically, most investors seem to have a mid-term investment time horizon of approximately five (5) years.

To estimate $E(R_m)$, we use past data of a stock index, as the S&P 500 or the FTSE 20. In this case, we assume that the average return of the stock index is the expected future market portfolio return. Alternatively, the analyst can estimate the potential market return with the corresponding probabilities that these returns will occur and then calculate the total expected return and their standard deviation.

To estimate the security's beta coefficient (β) (or the portfolio's beta coefficient) we calculate the security's (or portfolio's) returns (dividend plus capital gains) for various periods and the corresponding returns of the market index in the same periods. Beta is the slope of the characteristic line. The beta coefficient is usually estimated by using the ordinary least squares - OLS regression. In this case, we assume that the security's historical beta, estimated using the past data, will remain approximately the same in the future. However, the problem is that the estimates of beta coefficient vary depending on the number of the observations and on the market index.

Example 2

One year ago, your portfolio return was 20% and the beta coefficient was 1.2. During the same period, the market portfolio return was 16% and the return on the risk free asset was 6%. Evaluate the return of your portfolio.

Answer:

In order to evaluate the portfolio's return we should create a benchmark portfolio that will be completely diversified, consequently, it will contain systematic only risk. This benchmark portfolio is created by using the Capital Asset Pricing Model and contains the same systematic risk as our portfolio

risk, which is β= 1.2. Then we find the benchmark portfolio return and compare it to our portfolio return. The benchmark portfolio return is known as normal return. If our portfolio contains unsystematic risk, then its return will differ from the benchmark portfolio return. This difference between the portfolios' returns is known as abnormal return.

According to the Capital Assets Pricing Model, the benchmark portfolio normal return with systematic risk equal to our portfolio systematic risk, will be:

$$E(R_p) = R_f + [\ E(R_m)\text{-}r_f\ ] \times (b_p) \Rightarrow$$
$$E(R_p) = (0.06) + [\ (0.16\text{-}0.06) \times (1.2)\ ] = 0.18 \text{ or } 18\%$$

If we symbolize our portfolio abnormal return with (AR_p), our portfolio realized return with (RR_p)and the expected return (i.e the normal return) that our portfolio should have depending on the systematic risk with $E(R_p)$ (i.e the benchmark portfolio return), we will have

$$AR_p = RR_p - E(R_p) = 0.20 - 0.18 = 0.02 \text{ or } 2\%$$

Because of its systimatic risk our portfolio realized higher return than the expected.

Activity 2/Chapter 7

A year ago Mr. P. had invested 30% of his capital in stock A, 30% in the stock B, 20% in the stock C and 20% in the stock D. Information about these shares is given in the following table:

Stock	Price at the beginning of the year	Price at the end of the year	Dividend per share	Beta coefficient
A	€4.5	€5.8	€0.50	1.12
B	5.0	3.0	0	1.14
C	4.5	5.0	0.30	0.95
D	8.3	10.5	0.35	1.16

We know that one year ago, the market return (using as proxy the S&P 500 index) was 12% and that during the same period the risk free asset (using as proxy the return of the treasury bills) was 6%. Evaluate Mr. P.'s portfolio return.

7.6 EMPIRICAL TESTS OF THE CAPITAL ASSET PRICING MODEL

There are a lot of studies that empirically test the Capital Asset Pricing Model[3]. Researchers focus mainly in two questions. The first refers to the diachronic stability of the beta coefficient. In other words, is beta coefficient, based on past data, a good estimate of the future beta coefficient? The second question refers to testing if there is a positive linear relationship between beta coefficients and the returns on risky assets.

Most empirical studies regarding the diachronic stability of beta led to the following conclusions:

- The beta coefficients (β_i) of individual securities, estimated using past data, do not remain constant over time. Consequently, historical betas differ from the securities' future betas.
- The big portfolios beta coefficients (β_p) (that include e.g. 50 shares), estimated using past data remain constant over time, because the individual beta (i.e. of each security) variations are mutually excluded.

The empirical investigation of the relationship between systematic risk and the returns on the risky assets is usually made by estimating the coefficients of the following equation using past data and regression analysis:

$$E(R_i) = \alpha_1 + \alpha_2 \beta_i \qquad (7.3)$$

where, $E(R_i)$ is the security's i average return, for a number of periods, α_1 should approach R_f, α_2 should approach the market portfolio risk premium, and β_i is the estimate of the security i's beta coefficient.

Empirical studies have led to ambiguous conclusions regarding the above equation. Past studies led to the following conclusions:

- The security market line is linear with positive slope.
- α_1 is usually higher than the return of the risk free asset (i.e. R_f).
- α_2 (i.e. the slope of the estimated line) is lower than expected according to the compensation theory of the market portfolio, i.e. the term $[E(R_m)-r_f]$. In other words, security market line seems to be more horizontal than that forecasted from the Capital Asset Pricing Model.
- There are not enough findings which support that investors are compensated for the unsystematic risk that they undertake.

[3] *For more information see Elton, Gruber, Brown and Goetzmann (2003), pp.338-363.*

Recent empirical studies cast doubt about the positive linear relation between the beta coefficient and the return on the risky assets. Thus, empirical studies have led to ambiguous results until nowadays.

According to the above analysis, it is obvious that the Capital Asset Pricing Model may not lead to reliable conclusions. This may happen for two major reasons. First, most empirical studies examine realized returns, while the theory refers to expected returns. Second, the market portfolio differs from the Stock indices, used by the researches. As we have already mentioned, the market portfolio includes shares, bonds, gold, currencies, land, antiques, stamps, human capital and generally all the assets (domestic or not) that contain risk. Thus, a stock index is only a small part of the real market portfolio. Furthermore, Roll (1977) implied that in order to apply the CAMP we should first check: a) if the specific portfolio used as the market portfolio lies on the efficient frontier of Markowitz and b) if this portfolio is the real market portfolio (M). He proved that if the portfolio, used as the market portfolio, lies on the efficient frontier of Markowitz, then there is a linear relationship between return and beta. Thus, all studies do not test the CAMP, but examine if the portfolio used as market portfolio is efficient. Accordingly, Roll proved that in case we choose another portfolio we may get different results. That is why Roll implied that the CAMP has not been empirically proved yet and doubted if it can be empirically tested.

Note that Roll's critic to the model is that it has not yet been proved. However, this does not mean that the CAMP is false and that it should not be used in security valuation.

Synopsis

- **Capital market theory uses the Markowitz portfolio theory to analyze how various assets are priced in the market.**
- **Capital market line shows the trade-off between the expected return and the risk for efficient portfolios, assuming that the market is in equilibrium. This line shows that the expected return of an efficient portfolio is equal to the return on the risk free asset, plus the product of the market price of risk multiplied by the risk of the efficient portfolio.**
- **According to the CAMP approach, the efficient frontier is a straight line, which starts from the risk free asset and is tangent to the efficient frontier. The point that the straight line is tangent to the efficient frontier is the market portfolio and contains all risky assets.**
- **The security market line explains how securities and portfolios are evaluated in the market (either efficient or not). The security market line shows that the expected return of each asset is equal to the return on the risk free asset, plus a risk premium for the systematic risk that the investor undertakes when he buys the specific asset. This risk premium is higher as the systematic risk of the asset increases and is measured by the asset's beta coefficient. The formula of the security market line (SML), is: $E(R) = R_f + [E(R_m) - R_f] \times \beta_i$. This formula is also the formula of the CAPM.**
- **Most empirical studies of the CAMP conclude that the beta coefficients of each security, estimated using past data, do not remain constant over time, while the beta coefficient of big portfolios, do remain constant. Furthermore, empirical studies testing the potential positive linear relationship between the beta coefficients and the returns on the risky assets have led to ambiguous results. Finally, R. Roll implies that the model has not been proved because the market portfolio differs from the Financial Stock ratios used by empirical studies.**

Answers to Activities

Activity 1

Answer:

(a) According to the CAMP, we have:

$E(R) = R_f + [E(R_m) - r_f] \times (\beta_i) \Rightarrow \beta_i = [E(R) - r_f] / [E(R_m) - r_f]$. Therefore, we have:

For stock A: $\beta_A = [(0.16)-(0.06)]/[(0.12)-(0.06)] = 1.67$

For stock B: $\beta_B = [(0.08)-(0.06)]/[(0.12)-(0.06)] = 0.33$

In order to find the return's standard deviation of each stock we use the following equation:

$\sigma^2 = [\beta_i^2 \times \sigma_m^2] + \sigma_{ei}^2$. Therefore, we have:

For stock A: $\sigma_A^2 = [(1.67)^2 \times (0.20)^2] + (0.0650) = 0.1766 \Rightarrow \sigma_A = 0.4202$

For stock B: $\sigma_B^2 = [(0.33)^2 \times (0.20)^2] + (0.0755) = 0.0799 \Rightarrow \sigma_B = 0.2826$

(β) $E(R_p) = \Sigma w_i \times E(R) = [(0.80) \times (0.16)] + [(0.20) \times (0.08)] = 0.1440$ or 14.40%

$\beta_p = \Sigma w_i \times \beta_i = [(0.80) \times (1.67)] + [(0.20) \times (0.33)] = 1.4$

$\sigma_p^2 = [\beta_p^2 \times \sigma_m^2] + [\Sigma w_i^2 \times \sigma_i^2] = [(1.4)^2 \times (0.20)^2] + [(0.80)^2 \times (0.0650)] +$

$+ [(0.20)^2 \times (0.0755)] = 0.1230 \Rightarrow \sigma_p = 0.3507$

Activity 2

Answer:

The valuation includes four steps. First, we will calculate the return of each share contained Mr. P's portfolio.

$$r_A = (0.5+5.8-4.5) / 4.5 = 0.40 \text{ or } 40\%$$

$$r_B = (0+3-5) / 5 = -0.40 \text{ or } -40\%$$

$$r_C = (0.3+5.0-4.5) / 4.5 = 0.18 \text{ or } 18\%$$

$$r_D = (0.35+10.5-8.3) / 8.3 = 0.31 \text{ or } 31\%$$

Second, we will calculate the realized return of the total portfolio.

$$RR_p = \Sigma w_i r_i = (0.30)(0.40) + (0.30)(-0.40) + (0.20)(0.18) + (0.20)(0.31) \Rightarrow$$

$$RR_p = 0.1290 \text{ or } 12.90\%$$

Third, we will calculate the beta coefficient of the portfolio.

$$\beta_p = \Sigma w_i \beta_i = (0.30)(1.12) + (0.30)(1.14) + (0.20)(0.95) + (0.20)(1.16) = 1.1$$

Fourth, we create a benchmark portfolio, which will be completely diversified, containing only systematic risk. This benchmark portfolio is constructed by the CAMP and contains the same systematic risk to Mr. P's portfolio, i.e. $\beta = 1{,}1$. Then, we calculate the benchmark portfolio return and compare it to the portfolio return of Mr. P.

According to the CAPM the benchmark portfolio normal return with systematic risk equal to the systematic risk of Mr P.'s portfolio is:

$$E(R_p) = R_f + [E(R_m)\text{-}r_f] \times (b_p) \Rightarrow$$

$$E(R_p) = (0.06) + [(0.12\text{-}0.06) \times (1.1)] = 0.1260 \text{ or } 12.60\%$$

If we symbolize with (AR_p) the portfolio abnormal return, with (RR_p) the realized portfolio return and with $E(R_p)$ the expected return (i.e. the normal return) that the portfolio of Mr. P should have according to the systematic risk (i.e. the benchmark's portfolio return), we have:

$$AR_p = RR_p - E(R_p) = 0.1290 - 0.1260 = 0.0030 \text{ or } 0.3\%$$

Thus, Mr. P's portfolio realized only a minor higher return than the expected, depending on its systematic risk.

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CHAPTER 8

EVALUATION OF PORTFOLIO PERFORMANCE

The Scope of the Chapter

In this chapter, we help you understand how portfolio performance is evaluated. The evaluation of portfolio performance helps the investor determine if the formation and the management of a specific portfolio compensate for the time or/and the expenses spent for it.

Learning Objectives

When you have finished studying this chapter, you will be able to:

- mention two major requirements of a portfolio manager and how he/she can achieve them
- describe the Treynor measure
- describe the Sharpe measure
- explain the main difference between the Treynor measure and the Sharpe measure
- describe the Jensen measure
- describe how portfolio diversification can be measured
- mention two factors that affect the portfolio performance measures.

Key Words

- Treynor measure
- Sharpe measure
- Jensen measure
- Characteristic line in risk premium (or excess return form)

Introductory Comments

In previous chapters we examined how various securities are analyzed, evaluated and combined to form a portfolio, which corresponds to the investor's investment objectives. It is both expensive and time consuming to analyze and select securities for a portfolio. Therefore, the investor must determine whether this effort is worth the time and money invested in it. In case the investor builds and manages his/her portfolio by himself/herself and his/her investment choices do not result in satisfactory rates of return, he should consult a professional portfolio manager. In this case, he/she should determine if the portfolio return justifies the management costs, i.e. the costs paid to the professional manager. According to the above, it is obvious that the portfolio performance evaluation is a very important process.

This chapter includes seven sections. The first section analyzes some basic concepts of portfolio performance evaluation. The second section presents the

Treynor measure. The third section describes the Sharpe measure. The fourth section compares the Treynor measure with the Sharpe measure. The fifth section examines how portfolio diversification can be measured. The sixth section analyzes the Jensen measure. Finally, the seventh section refers to some problems that are related to the three measures of portfolio performance evaluation.

8.1 BASIC CONCEPTS OF PORTFOLIO PERFORMANCE EVALUATION

Portfolio management, either professional or private, has the following objectives:
- To derive above – average returns for a given risk class.
- To completely diversify the portfolio in order to eliminate all the unsystematic risk, relative to the benchmark portfolio.

The first objective can be achieved if the manager has an exceptional ability of choosing the “right” securities and/or an exceptional ability of timing the market. A portfolio manager that is able to select underpriced securities over time, he/she will achieve above – average returns for a given risk class. If this manager can “time the market” over a long period of time (i.e. from a fall to a rise and reversely), he/she will again achieve above – average risk-adjusted returns, because he/she will be able to change the portfolio composition before the market changes. The second objective can be achieved by choosing several different securities to be included in the portfolio. The level of portfolio diversification can be measured on the basis of the correlation between the portfolio returns and the returns of a market portfolio (or some other benchmark index). A completely diversified portfolio is perfectly correlated with the fully diversified benchmark portfolio.

According to the above analysis it is obvious that these two objectives of a portfolio manager are very important. If these objectives are not achieved, the management cost of the portfolio is not compensated for. This cost may have the form of monetary expense (i.e. the cost of hiring a professional manager), or the form of the lost time (if the investor manages his/her portfolio himself/herself). Thus, portfolio performance evaluation is exceptionally important for both professional portfolio managers and investors.

Until the beginning of the 1960s, most researchers classified the examined portfolios into similar risk classes (based on a risk measure, i.e the standard deviation of the portfolio return) and then compared the rates of return for alternative portfolios directly within these risk classes. However, after the creation of the Capital Asset Pricing Model (CAPM), several researchers suggest the use of some **composite (risk - adjusted) measures of portfolio performance**. These measures include both return and risk in portfolio performance evaluation. The most common composite measures are:
- the Treynor measure,
- the Sharpe measure and
- the Jensen measure.

8.2 THE TREYNOR MEASURE

Treynor (1965) developed the first composite measure of portfolio performance that included risk. He was interested in a measure of performance that would apply to all investors, regardless of their risk preferences. His composite measure of portfolio performance is the ratio of the risk premium (or additional portfolio return), divided by the portfolio's beta coefficient[1]. In other words, this measure calculates the risk premium of the examined portfolio, per unit of its systematic risk. The measure of Treynor is:

$$T_p = \frac{\overline{R_p} - \overline{R_f}}{\beta_p} \qquad (8.1)$$

where, $\overline{R}_p$ = the average rate of return for portfolio p during the examined time period, $\overline{R}_f$ = the average rate of return on a risk-free asset during the same time period, β_p = the portfolio's beta coefficient, and $\overline{R}_p - \overline{R}_f$ = the risk premium of portfolio p.

The higher the Treynor ratio of a portfolio, the better the portfolio return during the examined period. The Treynor ratio of the market portfolio is the slope of the security market line (SML). Consequently, if we compare the Treynor ratio of a portfolio with the corresponding market portfolio's Treynor ratio, the portfolio can be presented in the same diagram that depicts the security market line. If the Treynor ratio of the examined portfolio is higher than the Treynor ratio of the market portfolio, the portfolio will lie above the security market line, which means that during the examined time period it had superior return proportionally to its systematic risk. If the Treynor ratio of the examined portfolio is lower than the Treynor ratio of the market portfolio, the portfolio will lie under the security market line, which means that during the examined time period it had inferior return proportionally to its systematic risk. A graphic depiction of the portfolio's p Treynor ratio is presented in diagram 1. The Treynor ratio is the slope of the straight line, which connects portfolio p with the return on the risk free asset. The steeper the straight line, the higher its slope and the better the portfolio performance.

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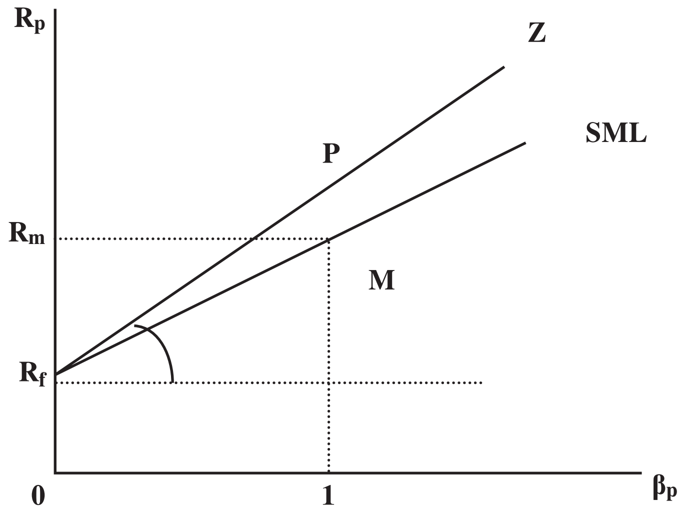
[1] *This measure is also known as reward-to-volatility ratio.*

Diagram 1
The Treynor measure



8.3 THE SHARPE MEASURE

Sharpe (1966) conceived a portfolio performance measure similar to the Treynor measure. However, he included the total risk of the portfolio by considering the standard deviation of the returns rather than considering only the systematic risk summarized by beta. His composite portfolio performance is the ratio of the risk premium (or additional portfolio return), divided by the standard deviation of the portfolio return[2]. In other words, this measure calculates the risk premium of the examined portfolio, per unit of its total risk. The measure of Sharpe is:

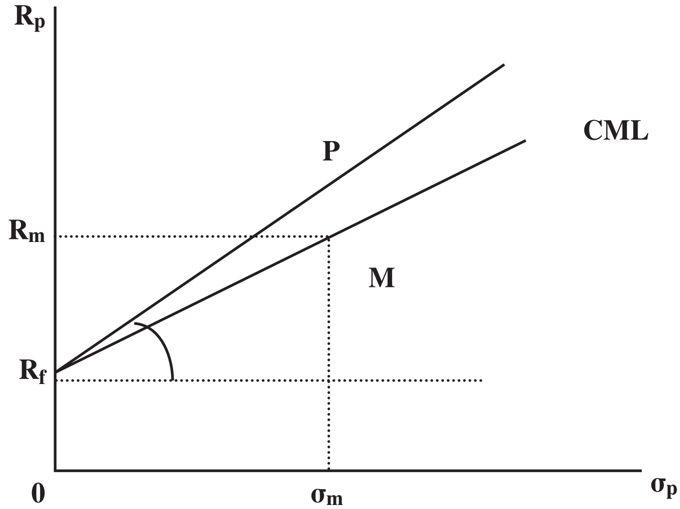
$$S_p = \frac{\overline{R_p} - \overline{R_f}}{\sigma_p} \qquad (8.2)$$

where, $\overline{R}_p$ = the average rate of return for portfolio p during the examined time period, $\overline{R}_f$ = the average rate of return on a risk-free asset during the same time period, σ_p = the standard deviation of the portfolio return during the examined time period and $\overline{R}_p - \overline{R}_f$ = the risk premium of portfolio p.

The higher the Sharpe ratio, the better the portfolio return during the examined time period. The Sharpe ratio of the market portfolio is the slope of the capital market line (CML). Consequently, if we compare the portfolio's Sharpe ratio with the corresponding Sharpe ratio of the market portfolio, the portfolio can be presented in the same diagram that depicts the capital market line. If the Sharpe ratio of the examined portfolio is higher than the Sharpe ratio of the market portfolio, the portfolio will lie above the capital market line which means that during the examined time period it had superior return proportionally to its total risk. If the Sharpe ratio of the examined portfolio is lower than the Sharpe ratio of the market portfolio, the portfolio will lie under the capital market line which means that during the examined time period it had inferior return proportionally to its total risk. A graphic depiction of the portfolio's p Sharpe ratio is presented in diagram 2. The Sharpe ratio is the slope of the straight line, which connects portfolio p with the return on the risk free asset. The steeper the straight line, the higher its slope and the better the portfolio performance.

[2] *This measure is also known as reward-to-variability ratio.*

Diagram 2
The Sharpe measure



8.4 TREYNOR VERSUS SHARPE MEASURE

The Treynor measure is similar to the Sharpe measure. They differ only in the way that they measure the portfolio risk. The Treynor measure uses the portfolio systematic risk, while the Sharpe measure uses the portfolio total risk. Thus, the Sharpe measure evaluates portfolio performance on the basis of both rate of return performance and diversification.

The question that arises is the following: If we classify various portfolios according to their return, will the above measures give the same rankings? The answer to this question is that the ranking depends on the rate of diversification of the examined portfolios. If a portfolio is completely diversified (i.e. a mutual fund), both measures will give identical rankings. However, if a portfolio is not well diversified, then the classification with the Treynor measure will be different from that of the Sharpe measure. This happens because a not well-diversified portfolio will have relatively higher standard deviation than beta coefficient. Thus, this portfolio will have lower Sharpe ratio, than Treynor ratio.

The above analysis shows that the choice of the performance measure depends on the examined portfolio. If the portfolio represents the total investment of an investor, the suitable measure is the Sharpe ratio. If the portfolio is a subset of the investor's total portfolio (i.e. the investor has also other portfolios), then the suitable measure is the Treynor ratio. Note that these two measures provide different but supplementary information and even though they are highly correlated, most researchers support that both should be used in portfolio performance evaluation.

8.5 MEASURING PORTFOLIO DIVERSIFICATION

The previous section (and previous chapters as well) referred to the importance of portfolio diversification. Portfolio diversification can be measured by the correlation between the portfolio returns and the returns for a market portfolio. This correlation is a by-product of the characteristic line estimation. Recall that the characteristic line is the regression line of best fit through a scatter plot of rates of return for the individual risky asset and for the market portfolio of risky assets over some specific past time period. While estimating the regression, we also calculate the square of the correlation coefficient, which is known as the coefficient of determination and is usually symbolized with R^2. The coefficient of determination shows the percentage of the total variance of the portfolio explained by the changes in the returns on the stock index. For example, if $R^2 = 0.90$ this means that the characteristic line interprets 90% of the total dispersion of the portfolio returns round their average value. The rest 10% is owed to other factors, included in the residuals.

The coefficient of determination lies between 0 and 1 [i.e. $0 \leq R^2 \leq 1$]. If the examined portfolio is completely diversified, then the coefficient of determination approaches one. In this case, the portfolio returns are completely determined by the index returns (which is used as a proxy of the total market). If the coefficient of determination of the portfolio is relatively low (i.e. $R^2 = 0.70$), then the portfolio returns are also affected by other factors (apart from the market), which could be eliminated through the process of diversification. In this case, the portfolio is exposed to unsystematic risk.

Activity 1/Chapter 8

The following information about five portfolios and a Stock Exchange index (the S&P 500) are given for a time period of 10 years.

Portfolios	Annual average return R_i (%)	Standard deviation σ_i (%)	Beta coefficient (β_i)	Coefficient of determination (R^2)
A	12	31	1,10	0,75
B	14	22	1,15	0,98
C	18	28	1,30	0,94
D	17	25	0,90	0,96
E	9	15	0,70	0,65
S&P 500	10	20		

The average rate of return on the risk free asset at the same time period was 6%. Answer the following questions.

(a) Classify the portfolios using the Sharpe and Treynor measures.

(b) Compare the ranking of portfolios A and B. Are there any differences in the ranking of these two portfolios? If there are any, explain why.

(c) Find which portfolios had higher risk-adjusted return than the market portfolio.

8.6 THE JENSEN MEASURE

Jensen (1968) suggested a performance portfolio measure, which is similar to the two previously mentioned measures, because it is also based on the Capital Asset Pricing Model[3]. The Jensen measure is the alpha portfolio value, which is determined by the difference between the realised portfolio return and its required return that corresponds to the portfolio's systematic risk. More specifically, the alpha portfolio value is estimated by regressing the additional evaluated portfolio returns to the additional returns of the market index. Then, we examine if the alpha value is statistically significant and if it is positive or negative. The Jensen measure is analyzed in the following paragraph.

It is known that according to the Capital Asset Pricing Model, the expected portfolio return is equal to:

$$E(R_p) = R_f + [E(R_m) - R_f]\,\beta_p$$

This equation is ex-ante. Assuming that this model is empirically valid, the above equation can be expressed in terms of realised rates of return (ex-post) as follows:

$$R_p = R_f + [R_m - R_f]\,\beta_p$$

The returns represented in the above equation are realised during the examined period. Subtracting the risk-free asset return from both parts of the equation, we have:

$$R_p - R_f = [R_m - R_f]\,\beta_p \qquad (8.3)$$

Equation (8.3) is an alternative form of the characteristic line and is usually called **characteristic line in risk premium or excess return form**. This equation denotes that the additional portfolio return (i.e. the risk premium) is equal to the portfolio's beta coefficient times a market risk premium. This equation can empirically be examined by estimating the following regression

$$R_p - R_f = \alpha_p + [R_m - R_f]\,\beta_p + \varepsilon_p \qquad (8.4)$$

where, α_p = a fixed term which is called alpha value and ε_p = the random error. If all portfolios are in equilibrium and the Capital Asset Pricing Model holds, the fixed term should be equal to zero ($\alpha_p = 0$). Therefore, portfolio's alpha value measures the manager's contribution to the examined portfolio, since it represents how much of the rate of return on the portfolio is attributable to the manager's

[3] *This measure is also called differential return measure or alpha.*

ability to derive above-average returns adjusted for risk. Regression (8.4) will lead to one of the following three cases:

- A positive and statistically significant alpha value (α_p) means that the portfolio manager achieved superior return (compared to the one that corresponds to the undertaken systematic risk) during the examined time period. This success is ought to the manager's exceptional ability to forecast market changes and/or to select securities.
- A negative and statistically significant alpha value (α_p) means that the portfolio manager achieved inferior return (compared to the one that corresponds to the undertaken systematic risk) during the examined time period.
- A not statistically significant alpha value (α_p) means that the portfolio manager achieved a return proportional to the undertaken systematic risk, during the examined time period.

Equation (8.4) can also have the following form:

$$\alpha_p = \overline{R_p} - [\overline{R_f} + (\overline{R_m} - \overline{R_f})\, \beta_p]$$

where the bars above the returns correspond to average returns of the examined time period. This equation denotes that the alpha value equals to the realised return minus the required return which corresponds to the undertaken systematic risk.

The Jensen measure calculates risk premiums in terms of systematic risk; thus, it does not directly consider the manager's ability to diversify his portfolio. This is a reasonable assumption in case we examine well-diversified portfolios, for example mutual funds. However, this assumption does not hold for all portfolios, since many of them may not be well-diversified.

The Treynor and Sharpe measures use the average rate of return of each variable for the entire examined time period. On the contrary, the Jensen measure uses each variable's additional rate of return for various subperiods of the entire examined period. For example, if we examine a portfolio return for a period of 10 years, we must estimate the additional rate of return for each variable (that is used at the regression) for each month or each semi-annual period or each year (according to the degrees of freedom we want to estimate). Consequently, the Jensen measure has the disadvantage of requiring more calculations. On the other hand, the Jensen measure has a calculating advantage, because the estimation of the characteristic line in risk premium provides both the portfolio manager's contribution (alpha value), and the estimation of the portfolio systematic risk (portfolio beta coefficient).

8.7 PROBLEMS REGARDING PORTFOLIO PERFORMANCE MEASURES

The previously examined portfolio performance measures have an important disadvantage. They are derived from the Capital Asset Pricing Model and consequently, they are also criticized. Without repeating the critic for the Capital Asset Pricing Model, we will report two fundamental points that influence the performance measures.

(a) False determination of the security evaluation. When using performance measures we assume that the securities included in the examined portfolios are evaluated according to the Capital Asset Pricing Model. If this assumption is wrong, the portfolio classification, derived from these measures, will also be false. Supposing for example, that investors can borrow at a higher rate than the risk-free asset, the security market line (SML) has a higher constant and a lower slope than the estimated one [and the capital market line (CML) is no longer a straight line]. In this case, the performance measures are in favour of the low risk portfolios.

(b) False determination of the market portfolio (benchmark error). The Capital Asset Pricing Model assumes the existence of a market portfolio. The problem arises in finding a realistic proxy for this theoretical market portfolio. R. Roll detailed the problem with the market proxy in several studies and pointed out its implications for measuring portfolio performance. He showed that if the proxy for the market portfolio is not a truly efficient portfolio, then the security market line using this proxy may not be the true security market line – the true SML could have a higher slope. In this case, a portfolio plotted above the estimated security market line (SML), could actually plot below the real security market line. This means, that the use of Treynor and Jensen measures (which include the market portfolio) is doubtful, regarding the ranking of the examined portfolios. Consequently, we cannot tell if Treynor and Jensen portfolio performance measures refer to the manager's skills or to the fact that the financial index used, is not a good proxy for the market portfolio, even if it is the best available. Moreover, the beta coefficient computed using the proxy, may differ from the one that would be estimated using the true market portfolio. In this case, the Treynor and Jensen measures can lead to false conclusions. The Sharpe measure seems to be better than the other two measures, because it is not directly related to the market portfolio. Furthermore, the portfolio classification according to the Sharpe measure has a rather high correlation (i.e. 0.94 – 0.97) with the rankings of the other performance measures. Yet, we cannot completely avoid the problem of using a market index as a market portfolio proxy. The problem with the Sharpe ratio appears at the last stage of the evaluation procedure, when we must select the benchmark portfolio, with which we compare the examined portfolios. Most times, a market index is used as the benchmark portfolio, which is the proxy for the market portfolio.

Activity 2/Chapter 8

We have the following information on five portfolios for a period of 10 years:

Portfolios	Alpha price (α_i)	Beta (β_i) coefficient	Coefficient of determination (R^2)
A	3.5	1.12	0.95
B	1.8*	1.21	0.98
C	2.0	0.96	0.92
D	1.9	0.77	0.85
E	1.2*	1.31	0.66

Note: The asterisk denotes alpha prices statistically different from zero, in a confidence level of 5% of a two-tail t test.

(a) Which portfolio returns are better explained by the market returns?

(b) Which portfolio has the highest systematic risk and which the lowest?

(c) Which portfolio has the highest total risk?

(d) If we use the Jensen performance measure, which portfolio has the highest risk-adjusted return compared to the market?

Synopsis

- **Portfolio performance evaluation is very important for professional portfolio managers and investors. After the creation of the Capital Asset Pricing Model (CAPM), several researchers suggested the use of some composite (risk - adjusted) measures of portfolio performance. These measures include both return and risk in portfolio performance evaluation. The most common composite measures are the Treynor measure, the Sharpe measure and the Jensen measure.**
- **The Treynor measure is the ratio of the risk premium (or additional portfolio return) divided by the portfolio's beta coefficient.**
- **The Sharpe measure is the ratio of the risk premium (or additional portfolio return) divided by the standard deviation of the portfolio return.**
- **Portfolio diversification can be measured by the correlation between the portfolio returns and the returns for a market portfolio. This correlation is a by-product of the characteristic line estimation. The square of the correlation coefficient is the coefficient of determination and is usually symbolized with R^2.**
- **The Jensen measure is the alpha portfolio value, which is determined by the difference between the realised portfolio return and its required return that corresponds to the portfolio's systematic risk. More specifically, the alpha portfolio value is estimated by regressing the additional evaluated portfolio returns to the additional returns of the market index. Then, we examine if the alpha value is statistically significant and if it is positive or negative. Therefore, portfolio's alpha value measures the manager's contribution to the examined portfolio, since it represents how much of the rate of return on the portfolio is attributable to the manager's ability to derive above-average returns adjusted for risk. If all portfolios are in equilibrium and the Capital Asset Pricing Model holds, the fixed term should be equal to zero ($\alpha_p = 0$). A positive and statistically significant alpha value (α_p) means that the portfolio manager achieved superior return (compared to the one that corresponds to the undertaken systematic risk) during the examined time period.**
- **There are two basic problems that affect the performance evaluation measures. These are: the false determination of the security evaluation and the false determination of the market portfolio (benchmark error)**

Answers to Activities

Activity 1

Answer:

The shape ratios [$S_p = (R_p - R_f) / \sigma_p$] of the portfolios are:

A	0.1935
B	0.3636
C	0.4286
D	0.4400
E	0.2000
S&P 500	0.2000

The Treynor ratios [$T_p = (R_p - R_f) / \beta_p$] of the portfolio are:

A	5.4545
B	6.9565
C	9.2308
D	12.2222
E	4.2857
S&P 500	4.0000

Consequently, the portfolio's ranking is:

Ranking	Sharpe measure	Treynor measure
1	D	D
2	C	C
3	B	B
4	E	A
5	A	E

(b) Portfolios B, C and D are almost completely diversified [since R^2 approaches one]; thus, their classification according to the two different performance measures is the same. Portfolio B lies in the third place regardless of which performance measure is used. Portfolios A and E are not well diversified [since R^2 is 0.75] and consequently, they are classified in different places. Portfolio A is fifth according to the Sharpe measure and fourth according to the Treynor measure. This is because Treynor measure uses only the systematic risk of the examined portfolio, whereas the Sharpe measure uses its total risk.

(c) Comparing the Sharpe and Treynor ratios of the five portfolios with the Sharpe and Treynor ratios achieved by the market index (S&P 500), during the same time period, we conclude that: (a) When we use the Sharpe ratio as a performance measure, all portfolios apart from A and E, have higher return than the market. Portfolio A, has lower return than the market, whereas portfolio E has the same return. (b) When we use the Treynor ratio as a performance measure, all portfolios have higher return than the market.

Activity 2

Answer:

(a) Portfolio's B returns are better explained by the returns of the market index used as a market portfolio proxy. This is because portfolio's B coefficient of determination (R^2 = 0.98) is the highest among the R^2's of the other portfolios.

(b) Portfolio E has the highest systematic risk (b_E = 1.31), whereas portfolio D the lowest (b_D = 0.77).

(c) Portfolio E has the lowest coefficient of determination (R^2 = 0.66) and consequently is less diversified than all other portfolios. As a result, portfolio E has not only the highest unsystematic risk, but also the highest systematic risk (b_E = 1.31). Providing we know that the total portfolio risk equals to the sum of systematic and unsystematic risk, we conclude that portfolio E has the highest total risk.

(d) Portfolio B has the highest positive and statistically important alpha (α_B=1.8). Consequently, this portfolio has the highest risk-adjusted return. Only portfolio E has a statistically important positive alpha (α_E = 1.2), but lower than that of portfolio B. The other portfolios' alpha values are statistically not significant and consequently we cannot be sure (at 95%) that they differ from zero. Therefore, the other portfolio managers achieved the expected return, i.e. the average return adjusted to the undertaken risk.

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CHAPTER 9

THE EFFICIENT MARKET HYPOTHESIS

The Scope of the Chapter

In this chapter, we help you understand some basics of the efficient market hypothesis as well as the effect of efficient markets on investment analysis and portfolio management. If markets are efficient, technical and fundamental analysis have ambiguous results.

Learning Objectives

When you have finished studying this chapter, you will be able to:
- define the efficient market hypothesis
- mention the factors that contribute to an efficient market
- distinguish the three forms of efficient markets and their most important empirical results
- mention the six anomalies of efficient markets
- explain the consequences of efficient markets on technical and fundamental analysis.

Key Words

- Efficient Market Hypothesis (EMH)
- Weak form of EMH
- Random walk hypothesis
- Semi-strong form of EMH
- Strong form of EMH
- Event studies
- Filter rule
- Market anomalies
- January effect
- Size effect
- Weekend effect

Introductory Comments

In previous chapters we saw how the market evaluates shares. All these theories have been developed under the assumption that there is an efficient relationship between the expected risk and return of all the investments. So far so good, but in order to check if markets are efficient we have to examine how they are organized, operate and react to new information. In case the market reacts very fast when new information arrives, then the prices are expected to reach the new equilibrium level very fast. On the contrary, if the new equilibrium level is reached slowly then arbitrage opportunities will show up. From the above analysis it is obvious that there is a direct relationship between market efficiency theory and how markets react when new information arrives. This direct relationship is the main topic of this chapter. This specific topic has created one of the major conflicts in the financial community. This conflict has great impact on professionals who deal with investment analysis and take portfolio management decisions.

If markets are efficient then the jobs of the professionals in the financial sector are in danger. Accordingly, no professional is willing to accept that the markets operate under complete efficiency.

This chapter includes four sections. The first section examines the basics of the efficient market hypothesis. The second section analyzes the three forms of market efficiency. The third section presents the most important empirical findings related to these theories. Finally, the fourth section describes certain cases that set the efficient market hypothesis into question.

9.1 BASICS OF MARKET EFFICIENCY

During the previous chapters where various financial instruments were analyzed, we saw that investors determine the intrinsic value of all financial instruments based on their forecast of the future financial inflows that they expect to obtain from each investment in each financial instrument. These forecasts are based on the information that the investors have. Hence, information has an important impact in the assessment and evaluation of financial investments by investors. When new information arrives in the market, investors react by buying (or selling) stocks. This investors' action, caused by the new information in the market, affects share prices. If this price adaptation happens very fast and accurate then the market is said to be efficient. Thus, a **market** is called **efficient** when stock prices adjust fast and precisely when new information arrives and consequently current stock prices completely incorporate all the known information. Even though the efficient market hypothesis is very general and refers to all markets, the majority of analysts believe that it can be easily applied in the Stock Exchange markets. For this reason, in the remaining chapter when we refer to markets we mean stock markets.

For a market to be efficient the following must hold:

- There must be a considerable number of investors that trade in the market and aim to maximize their wealth. All investors are supposed to analyze and evaluate stocks and act rationally and independently.
- New information is distributed to all investors at the same time and without cost.
- Information is supposed to be distributed in the market accidentally and all peaces of information must be, diachronically, independent from each other.
- As new information arrives in the market, investors are supposed to react fast and accurately. That causes stock prices to adjust to the new information. These price adjustments may be incomplete (for example, certain adjustments can be larger than they should, while other smaller) but unbiased (i.e. no one can forecast what kind of adjustment is going to happen).

All the above assumptions lead to the conclusion that changes in stock prices are independent and occur accidentally.

9.2 FORMS OF EFFICIENT MARKETS

The efficient market hypothesis (EMH) is the theory that refers to the efficient markets, and is usually divided in three categories depending on the type of the information that is incorporated in the stock[1]. These categories are the following:

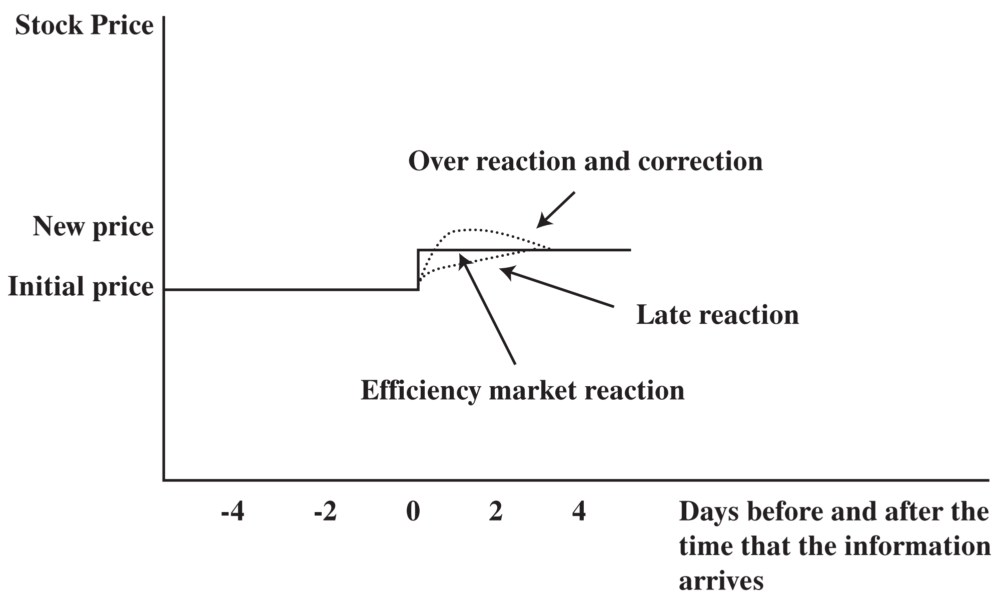
Weak form of market efficiency (EMH). This form of market efficiency assumes that stock prices fully reflect all security market information including past stock prices, rates of return, trading volume data and other information such as block trades, specialists' transactions. Under this hypothesis no investor based on current and past market data can achieve considerable gains. In this case, technical analysis does not play any important role. This weak form of efficiency is not identical but similar to what is known as **random walk hypothesis**. According to this approach, the successive returns from the stocks' investments (i.e. the changes in stock prices) are linearly independent from each other, and their probability distributions are constant over time (i.e. they never change). The weak form of market efficiency does not assume that the investments' returns are independent neither that the distributions of their probabilities are the same over time. This means that a correlation in the returns is possible and consequently the information included in past data and can be used to forecast future returns. The only thing that the weak form of market hypothesis assumes is that investors cannot exploit this information (i.e. the correlation among the returns) and are not able to receive returns greater than those that correspond to the risk of the particular investment. Thus, the random walk hypothesis sets more restrictions than the weak form of efficiency does. As a result, if the random walk hypothesis holds, then the weak form of market hypothesis holds too, but not the other way around. In order to explain the random walk hypothesis suppose that there is a roulette which has a variety of returns written on it. The next period's return is determined by the place that the ball will randomly stop. The roulette's results are not time dependant and past returns are not related to the future ones. Additionally, the roulette turns again and gives a new return for every new period, which means that the returns from one period to the other have the same probability distribution.

The semi-strong form of market efficiency. The market with this form of efficiency assumes that stock prices incorporate all public information. By saying public information we refer not only to the information that can be found in the

[1] *For more information about the forms of efficient markets see Fama (1970).*

past data of the stock market but also to the public information like balance sheets, profit and loss accounts, dividend statements, stock price ratios like the price to earnings per share (P/E), dividend payout ratios, split announcements, investments in new products, financial difficulties, economic news, political news etc. Thus, the semi-strong form of market efficiency includes all the assumptions of the weak form of market efficiency. According to the semi – strong EMH, stock prices adjust rapidly to the release of all public information by the time that this information is published. In this case both fundamental and technical analysis cannot provide new information. Diagram 1 presents the hypothetical reaction of stock prices when the new information arrives in the market when the market is efficient and when it is not.

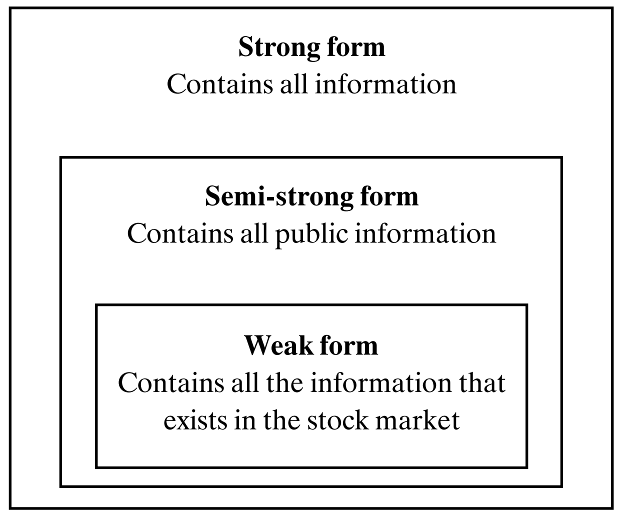
Diagram 1
Stock price reaction in new information



Strong form of the efficient market hypothesis. Under the strong form of market efficiency stock prices incorporate all information published or not (i.e. both public and private information). Thus, the strong form EMH includes both the weak and the semi-strong forms of efficiency. Under the strong form of efficiency no investor can have any access in information that can affect stock prices. Accordingly, no investor can acquire return greater than the normal.

Diagram 2 presents these three forms of market efficiency with the corresponding information that each form contains.

Diagram 2
Forms of Market Efficiency



9.3 EMPIRICAL INVESTIGATION

The studies that investigate the validity of the efficient market hypothesis test if investors, irrespective of their type, can systematically achieve returns higher than those which correspond to the undertaken risk (i.e. abnormal returns). This does not mean that markets do not delay to react when new information arrives and that they incorporate this information in stock prices and evaluate the stock in its intrinsic value. Moreover, this does not mean that there is no probability that a group of certain investors can have abnormal returns from their short-term investments. The only thing that the market efficiency theory supports is that investors cannot use the divergence of the stock prices from their intrinsic values in order to obtain abnormal (positive) long-term returns. The empirical studies can be divided in three categories, depending on the form of the market efficiency that they test[2].

Weak form of market efficiency. In order to test if the weak market form holds, we either use statistical tests of independence or we test the technical trading rules:

(a) **Statistical test for independence**. Researchers use these types of tests to check the hypothesis that changes in prices are time independent. Most researchers apply two types of tests: the autocorrelation tests and the runs tests. The autocorrelation test measures the correlation between the percentage changes in prices for various time intervals (e.g. one day, four days, nine days etc). The runs test denotes each positive change in stock prices with (+) (i.e. an increase in its price) and each negative change with (-). The result is a panel of positive and negative signs. If two or more consecutive signs are the same, then these signs constitute a "run". Thus, a run of positive signs is followed by a run of negative signs etc, and all these runs constitute a series. In this case the test of independence includes the comparison of the number of these runs with the expected number of runs that is given from a table of random numbers. In case there are not a lot of changes in the positive and negative signs (and consequently less runs) in the examined series from those expected randomly (i.e. expected from the table of random numbers), then there is a positive relation between the changes in stock prices. The majority of the empirical studies that apply tests of linear autocorrelation and runs tests, conclude that the current return of the share has a small positive correlation with the yesterday's return. However, this correlation is on average small and often negative for certain individual shares. That means that no investor can make

[2] *For more information see Elton, Gruber, Brown and Goetzmann (2003), pp. 402-443.*

use of the small statistically significant correlation that exists in the stock prices, because of the transaction costs paid when trading the share. Thus, these empirical studies conclude that the weak form of efficient markets does exist. At this point we should stress out that a recent empirical study by the Athens Stock Exchange led to the conclusion that its stocks do not follow the hypothesis of the random walk[3]. Thus, according to the efficient market hypothesis the Athens Stock Exchange is a non-efficient market, that means that the prices of stock traded in this market tend to move in a systematic way over time.

(b) Tests of trading rules. The majority of the empirical studies regarding this type of tests compare a specific technical trading rule with a buy-and-hold policy. The rule that is most commonly used is the filter rule, where the stock is bought (or sold) when the price change exceeds a "filter" value set by the analyst. Suppose that an analyst has set the filter at 5%. If the stock price increases at least by 5% from a specific base (i.e. from the previous lower price), then the stock should be bought and hold until the price of the stock decreases at least by 5% from a new base (i.e. from the previous highest price). In this case this stock should be sold, until the stock price will increase at least by 5% from the previous lower price. This process will continue each time that the price changes by 5% or more. The majority of the empirical studies lead to the conclusion that the technical trading rules have worse results against a buy and hold policy and consequently support the existence of the weak form of the efficient market hypothesis. However, we should point out that these empirical studies examine only simple technical rules and their samples include mainly big and known companies whose trade is exceptionally active. Thus, these studies may be biased.

Semi – strong form of market efficiency. We can test the existence of the semi – strong form of market efficiency with the following two ways:

(a) Event Studies. These empirical studies examine how fast the stock prices adjust to specific significant economic events (such as for example a stock split, a new equity issue, a change in the assets of the firms, a dividend announcement, significant changes in the economy or in the market etc). Moreover, these studies examine if there are abnormal returns immediately after the announcement of an event. The hypothesis of semi-strong market efficiency says that the stock price reacts, before or instantaneously at the time of the announcement, in such a way that no investor can obtain abnormal returns from its investment. The results from the majority of these studies support the existence of the semi – strong form of market efficiency.

(b) Studies that try to forecast future rates of return on an individual stock (or the return from the total market) by using information that is not included in the past data of the stock market (like the dividend yield, ratios such as the P/E,

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[3] *See Dockery and Kavussanos (1997).*

stock price to intrinsic value etc). These studies normally use time-series or cross-section analysis in order to investigate the stocks' return. Most of these studies doubt the existence of the semi-strong form of markets efficiency.

Note that when calculating the abnormal return, we also have to consider the change in the total market return for the same period. Before 1970, the empirical studies that calculated the abnormal rate of return used the following equation:

$$AR_{it} = R_{it} - R_{mt}$$

where, AR_{it} = the abnormal rate of return on stock i during period t, R_{it} = the actual rate of return on stock i during period t, R_{mt} = the market rate of return during period t.

However, specific stocks are more sensitive when the market portfolio changes, while other stocks are not that sensitive. For this reason, recent studies replaced the market index return with the expected return, calculated by the Capital Asset Pricing Model (CAPM) or any other relative model. Thus, the abnormal return is calculated by the following formula:

$$AR_{it} = R_{it} - E(R_{it}) \qquad (9.1)$$

where $E(R_{it})$ = the expected rate of return on stock i during period t.

If the market is efficient, then the actual return has to be equal to the expected return. In this case any test concerning the efficient market should first determine the benchmark model which is used to calculate the expected return. If the tests fail to verify the above equality, then either the semi-strong form of market efficiency does not hold, or the used model is not the correct one for this market (or both). Empirical studies for the Greek market (until now) do not prove that the semi-strong form of market efficiency holds.

Strong form of market efficiency. In order to test if the strong form of market efficiency exists we consider two distinct groups of investors. The first group includes all the corporate insiders and the second the portfolio managers. The common characteristic of these two groups is that both have access to unpublished information.

(a) **Corporate Insiders**. This category includes large shareholders (i.e. investors that possess more than 10% of the total shares of the firm), the management of the firm, the financial executives of the firm, etc. All the above insiders have access to unpublished information of the firm (i.e. inside information). The majority of empirical studies that tests for abnormal returns from corporate insiders proved that this group of investors can obtain extra rates of return. These results show that the strong form of market efficiency does not exist due to the monopolistic access that these investors have to inside information.

(b) **Portfolio managers**. Many researchers consider that portfolio managers, especially mutual funds managers, possess extra information; actually they have the new information before it is known to the public. If this assumption holds, then examining the return of the portfolio manager can be considered as a test concerning the strong form of market efficiency. The majority of empirical studies supports that most of the portfolio managers are unable to obtain abnormal return over time. In fact these portfolio returns proved to be inferior compared with a simple buy and hold strategy. Thus, most empirical studies, concerning this category, support the existence of the strong form of market efficiency.

The general conclusions from the results of the above studies are the following:

- The majority of empirical studies supports the existence of the weak form of market efficiency.
- Most of the empirical studies regarding the semi-strong as well as the strong form of market efficiency, provide us with ambiguous results. Some of the studies support the existence of a semi-strong or strong form of efficient market while some others reject it.

All the above results doubt the usefulness of technical analysis, which is based on the aspect that the information is slowly diffused to investors and consequently the stock prices do not adjust to this information immediately but progressively. Accordingly, technical analysis supports that stock prices move based on market tendencies and that these tendencies create various patterns which can be formed. This view is completely opposite to the efficient market theory (even in its weak form). If capital markets are efficient, then stock prices incorporate all information that can be extracted from the data of the stock market and there is no technical rule that can create abnormal returns using historical data, given that the risk and the transaction costs are known. This happens because technical analysts act when the information is published, while the value of the information has already been taken into account in an efficient market.

The fundamental analysis considers that each share has an intrinsic value which can be determined by examining various factors that influence this intrinsic value, for example the current and future profits of the firm, the risk of the firm and the market interest rates[4]. If the intrinsic value of a share is higher (lower) from its current market price, fundamental analysis suggests to buy (sell) this particular share. This suggestion is based on the opinion that the difference between the two prices will finally be eliminated, when the market will realise that the current value of the share is not the same with its intrinsic value and it will be corrected in the future by the market itself. Fundamental analysis usually starts with a historical analysis of the financial statements of the examined firm and a comparative analysis of the firm with all the other firms of the industry. This analysis is followed by an evaluation of the firm's management team and the analysis of the total prospects of the economy and the industry as well. If markets are efficient and stock prices incorporate all public information, then the effectiveness of fundamental analysis is doubtful. As fundamental analysts base their estimations on public information, their estimates

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[4] *The way by which the intrinsic value of a stock is determined, is presented in the chapter "Stock Valuation".*

will not differ from the estimates of their competitors. Even when an analyst discovers a company with good prospects, it will not offer him any extra rate of return, because the same company is known to the rest of the analysts as well. In this case an analyst will only be rewarded if he discovers a company that have better prospects than those that the other analysts discovered. Alternatively, the analyst can be rewarded if he discovers a company with bad prospects, but not as bad as the total market expects. From the above it is obvious that a fundamental analyst can yield extra returns only to the extend that some other analysts can not perform a better analysis than he does.

9.4 MARKET ANOMALIES

There are certain empirical findings which appear to be contradictory to the efficient market hypothesis and more specifically to the semi – strong form. These findings are known in the relative bibliography as market **anomalies** and, until now, there is no satisfactory explanation about this phenomenon. Some of these cases are[5]:

Stock price to earnings per share ratio (P/E). Stocks with low P/E ratio outperform high P/E stocks. The same results appear even with adjustment for firm size, industry effects and risk.

Market Value to Book Value Ratio. Shares with low MV/BV ratio outperform high MV/BV stocks.

The Size effect. It appears that small firms have larger risk-adjusted returns than larger firms.

The January anomaly. It has been observed that stock prices tend to decrease during the last days of December and to increase during the first days of January. Most of the researchers who analyse the January effect in the USA claim that this anomaly has an explanation. More specifically, they believe that portfolio managers try to realize losses so as to pay fewer taxes. If this hypothesis is correct, then portfolio managers should sell at the end of the year the stocks with current prices lower than the buying prices. According to this strategy, portfolio managers will either buy (at the beginning of January) again the stocks that they sold during the last days of December or rearrange their portfolios and buy new shares.

Unexpected Earnings. It has been found that the market reacts with a delay at the "positive earnings' surprises" included in the firm statements concerning their quarter profits. Moreover, there is a significant relationship between the unexpected earnings' size that a company announces and the change in the stock price after the announcement.

Weekend effect. It has been observed that stock prices tend to decrease on Monday more often than on any other day of the week. This negative result in stock return has no certain economic explanation and is called weekend effect.

Activity 1/Chapter 9

Which is the relationship between the weak form of EMH market and technical analysis?

[5] *For more information see Elton, Gruber, Brown and Goetzmann (2003), pp. 402-443.*

Activity 2/Chapter 9

You meet a professional portfolio manager who complains about not having an analyst with exceptional abilities. How would you consult him to manage his portfolio if you believe that markets are efficient?

Synopsis

- **A market is efficient when stock prices adjust fast and precisely when new information arrives and consequently current stock prices incorporate completely all the known information. The efficient market hypothesis (EMH) is the theory that refers to the efficient markets, and is usually divided in three categories depending on the type of the information that is incorporated in the stock. These categories are the weak form, the semi-strong form and the strong form of market efficiency.**
- **The weak form of market efficiency assumes that stock prices fully reflect all the security market information.**
- **The semi-strong form of market efficiency assumes that stock prices incorporate all public information.**
- **The strong form of market efficiency assumes that stock prices incorporate all information, published or not (i.e. both public and private information)**
- **The findings of most empirical studies support the existence of the weak form of market efficiency. Most empirical studies provide contradictory results regarding the existence of the semi-strong and the strong form of market efficiency.**
- **Market anomalies are certain empirical findings which appear to be contradictory to the efficient market hypothesis, especially to the semi – strong form. Some of these cases are the Price to Earnings per share ratio (P/E), the Market Value to Book Value Ratio (MV/BV), the size effect, the January anomaly, the unexpected earnings and the weekend effect.**

Answers to Activities

Activity 1

Answer:
In the weak form of EMH security prices incorporate all the information extracted by the security market. Consequently, as soon as new information concerning the security market arrives, stock prices in the efficient market incorporate this information quickly and precisely. On the other hand, technical analysis is based on the fact that information is diffused to investors slowly, and therefore stock prices do not adjust immediately, but progressively. Thus, technical analysis supports that we can formulate stock price fluctuations. This point of view is completely opposite to the efficient market theory (even in its weak form). In case risk and transaction costs have been taken into account and if a market is efficient, technical analysis cannot use past market data and create abnormal returns. This is because technical analysis takes place after the information announcement, the value of which, has already been considered by the efficient market. Consequently, when a market is efficient, technical analysis has no value.

Activity 2

Answer:
The given advices would be the following three:

First, the manager should determine his customer's preferences concerning risk and return. Afterwards, he should propose a portfolio that would approach the preferred risk and return. Since most empirical studies have found that portfolio beta coefficients remain constant over time, the above activity should not be difficult. Furthermore, the risk level can be regulated (increased or decreased), by investing part of the portfolio in government securities (i.e. in risk-free assets) and in risky securities (i.e. stocks). The portfolio part that has been invested in government securities can be altered in time, depending on the customers' preferences regarding risk.

Second, the portfolio should be completely diversified. Portfolio's systematic risk elimination denotes that this portfolio will have high positive correlation with a financial index (i.e. the market portfolio).

Third, the manager should minimize transaction costs (i.e. commissions, taxes etc.) by decreasing the security transactions.

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CHAPTER 10

TECHNICAL ANALYSIS

The Scope of the Chapter

In this chapter we help you understand some basic concepts regarding technical analysis. Particularly, technical analysis examines past market data to estimate future price trends and, therefore, to help the investor in his investment decision.

Learning Objectives

When you have finished studying this chapter you will be able to:
- tell the difference between technical and fundamental analysis
- mention the underlying assumptions of technical analysis
- mention the advantages and disadvantages of technical analysis
- describe specific technical trading rules
- explain how technical analysts use the trading volume.

Key Words

- Trends
- Technical rules
- Price patterns
- Self-fulfilling prophecies
- Contrary opinion rules
- Follow the smart money
- Market sentiment techniques
- Stock price and volume techniques
- Round lot
- Odd lot
- Margin debt
- Short interest rate

Introductory Comments

In previous chapters we examined how different types of securities are analyzed and evaluated. This chapter briefly describes a different way of security valuation and choice, using technical analysis. Although this method can be applied in the analysis of any security, conventional technical analysis focuses mainly on stocks.

Technical analysis is the oldest valuation method. Contrary to the fundamental approach, technical analysis does not need any financial data of the company or the industry. Technical analysts (or "technicians") believe that past market data such as prices and the trading volume can lead to an estimate of future price trends. Consequently, a technical analyst studies stock prices and trading volumes for a relatively long period to decide on his investment pattern.

This chapter includes three sections. In the first section we analyze the main concepts of technical analysis. In the second we present its advantages and disadvantages, and finally, in the third section we briefly present various technical rules that most technical analysts follow.

10.1 BASIC CONCEPTS

Technical analysis is the method which involves the explanation of past market data to forecast future price trends. This method can be applied either for each security, or for the entire market. Market data include information about daily stock prices, the trade volume, the prices of some financial indices as well as any other information reported in the market. The aim of the technical analysis is to forecast the short-term price stock fluctuation and not the exact price.

The main assumptions of technical analysis are[1]:

- Market prices are determined solely by the interaction of supply and demand.
- Supply and demand are influenced by several factors. These factors include all the economic variables relied on the fundamental analysis, as well as other variables governed by the market psychology, such as opinions, expectations, moods and guesses. The market continuously and automatically weighs these factors.
- Disregarding minor fluctuations, stock prices tend to move in **trends**, which persist for long lengths of time. This happens because information, which alters the supply and the demand, does not come to the market at one point in time but, rather, enters the market over a period of time. New information is earlier provided to some investors than to others. As various categories of investors receive new information and buy or sell a security accordingly, stock prices move gradually toward the new equilibrium. Therefore, stock prices are gradually and not immediately adjusted from an old equilibrium to a new one. This adjustment is expressed as a trend, which lasts until the stock price has reached a new equilibrium.
- The fluctuations in stock prices change in reaction to shifts in supply and demand relationships. Technical analysts are not interested in the reasons of the changes in supply and the demand. They believe that these changes can be estimated by observing how the market reacts. However, note that the analyst does not attempt to predict the new equilibrium value. He only needs to determine the starting point of the fluctuation and then to proceed to the necessary procedures. If the trend's time duration is particularly short, the procedure is actually worthless. Otherwise, the investor will probably be able to realize significant gains.

[1] *For more information, see Levy (1966), p. 58-63.*

10.2 ADVANTAGES AND DISADVANTAGES

The most important advantages of technical analysis are:

- According to technicians, a fundamental analyst will achieve higher rates of return only in case he receives new information before them and process it correctly and quickly. If these assumptions hold, the fundamental analyst will achieve a higher rate of return than the technical analysts, because the latter will act after stock prices will have adjusted to the new information. However, technicians claim that the above assumptions do not frequently hold.
- A great advantage, according to technical analysts, is that they are not based on the financial analysis of accounting statements. According to them, accounting statements have the following problems: (a) they do not include much information which is essential for the stock analysis, such as information related to sales and general expenses, or information about the profits per production line or per customer, (b) there are many ways of preparing an accounting statement, which can produce different values for expenses, income, return on assets etc.; all these ways are equally accepted. Consequently, an investor may have trouble comparing the accounting statements of different firms, especially in case they come from different industries, (c) many factors are difficult to quantify and are not included in the accounting statements. Examples include customer goodwill, employee training and loyalty etc.
- Technical analysts claim that they are more capable in determining the correct stock buy or sell moment. A technician will select the most appropriate moment, because he will wait until the stock starts falling or rising.

The most important disadvantages of technical analysis are the following:

- If markets are efficient, even at a weak form, technical analysis is actually worthless, because stock prices incorporate all public information concerning the financial market (as the weak form of Efficient Market Hypothesis implies), and thus it is impossible to forecast future stock prices, based on past prices. Consequently, only the “unexpected information” can influence stock prices, since the “expected information” has already been taken into consideration. The “unexpected” information cannot be forecasted and therefore stock price fluctuations cannot be forecasted as well. In this case, technical analysis is worthless.
- Certain past **price patterns** may not be repeated in the future. Consequently, a method that has used these patterns in the past and has been efficient may not lead to the same good results in the future as well. “Self-fulfilling prophecies” are included in the same category. Let’s assume, for example, that many analysts expect a stock selling at €2.5 a share to go to €3, if it should rise above its current

pattern and "breaks through" the technical channel of €2.7. As soon as it reaches €2.7, technical analysts will buy it and consequently the price will rise up to €3 as forecasted.

- If investors frequently use a technical rule, the rule will most probably be gradually worthless in the future.
- Technical analysis is based, to a large extent, on subjective estimations. Thus, technical analysts may reach to different conclusions and consequently to different investment decisions, conducting an analysis of the same data.
- The quantitative values of technical rules continuously change.

10.3 TECHNICAL TRADING RULES

Technical analysts use several different **technical trading rules**. However, the problem is that these rules may lead to more than one explanation. The most important technical rules are included in one of the following four categories:

- Contrary opinion rules.
- Follow the smart money.
- Market sentiment techniques.
- Stock price and volume techniques.

In this chapter, we will briefly examine the first three cases[2].

10.3.1 Contrary opinion rules

Most technical analysts believe that the majority of investors usually have a false opinion about the total market. Thus, these technicians trade in the opposite direction from the majority of investors and apply the contrary opinion rules, some of which are described in the following paragraphs.

1. Odd-lot theory

Typically, stocks are usually traded at items of 10 or multiples of 10 or even at items of 100 or multiples of 100, depending on the stock exchange policy. This amount is called **round lot**. Odd lot is the stock amount which is smaller than the round lot. According to the odd-lot theory, small investors who buy and sell odd lots frequently, are supposed to make false choices regarding the forecast of the market price fluctuations. These investors buy (or sell) when the market is at its highest (lowest) level. Technical analysts take advantage of those false choices by using various indices, such as the odd-lot index and the odd-lot short sales index.

2. Investment advisory opinions

This strategy is based on the estimation of the bearish sentiment index. This index is calculated as the number of investment advisors with negative opinions divided by the total number of investment advisors. As this index increases, the negative opinion about the prospects of the market also increases. Supporters of the opposite opinion will follow the opposite direction of the bearish sentiment

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[2] *For more information, concerning the four technical rules, see Reilly and Brown (2003), pp.625-651.*

index. Investors and investment companies which follow such techniques, tend just to follow trends and not to forecast them. Consequently, their estimations are false according to the supporters of the opposite side (at least regarding the highest and lowest market levels).

3. Mutual fund cash positions

Several technicians consider mutual funds as small investors that buy or sell odd lots. Thus, they believe that mutual funds make false estimations about future price fluctuations when the market is at the highest or at the lowest level. Consequently, mutual funds are expected to preserve high (low) liquid funds when the market is at its lowest (highest) level. Generally, when the cash to total assets ratio for mutual funds is lower than 7%, this means that mutual funds make positive forecasts about future market price fluctuations, whereas if it is higher than 12% they make negative forecasts about future trends in prices.

The quantity of liquid assets of mutual funds is an indication of their potential purchasing power. Regardless of how liquid assets of mutual funds were created, in case they are many, they should be invested. In this case, stock prices will increase.

4. The put/call ratio

Many technicians support that investors that participate in the option market usually lose a part of their investments. Investors that buy call options expect that stock prices will increase, while those who buy put options expect that stock prices will decrease. The put/call ratio usually fluctuates between 0.70 and 1.00, because most investors are optimistic. If, for example, the ratio is 0.80, we know that 80 put options are traded per 100 call options.

10.3.2 Follow the smart money

Some technical analysts create rules, based on sets of specific indicators, to follow the behavior of smart, sophisticated investors. This policy is called "follow the smart money" and includes the following rules:

1. The confidence index

This index is published by the Barron's magazine every week and estimates the investor optimism or pessimism by examining their trade in the bond market. The confidence index is the ratio of average yield on 10 top-grade corporate bonds divided by the yield on the Dow Jones average of 40 bonds. Therefore, this index measures the difference in yield spread between high-grade bonds and a large cross section of bonds. Because the yields on high grade bonds should always be lower than those on a large cross section on bonds, the confidence index is always smaller than one (1). As investors become more optimistic, they buy more bonds of lower quality, for the added yield, which causes a decrease in the average yield for the large cross section of bonds relative to the yield on high grade bonds. Thus, the conference index will increase. However, note that the above interrelation assumes that changes in the yield spread are caused almost exclusively by changes in the investor demand for different quality bonds. This is not always true. Changes in the supply of bonds can also lead to changes in yield differences.

2. Short sales by specialists

According to this method, technical analysts try to track the investment movements of specialists or market makers. In this case, they use the ratio of the short sales by specialists divided by the total short sales. A ratio lower than 30% provides a positive signal about price increases, while a ratio higher than 50% shows that prices will most probably fall. Note that this ratio is mainly used for short-term investments and that there is a time lag regarding the announcement of information about the short sales of specialists.

3. Debit balances in brokerage accounts (margin debt)

Debit balances in brokerage accounts represent borrowing by investors from their brokers. These investors are sophisticated and smart investors who decide to engage in margin transactions. They are assumed to possess special information. Thus, an increase in debit balances of brokerage accounts is considered a positive signal that prices will increase, while a decrease is considered a negative signal.

10.3.3 Market sentiment techniques

There are specific methods according to which technicians try to "perceive" the market sentiment. The following technical trading rules are included in this category:

1. The breadth of market

This technical rule is also called **advance - decline line** and shows the difference between the number of stocks that advanced in price and the number of stocks that declined and then adding this value to a cumulative total.

The common method of using this rule is by comparing the advance – decline line (or breadth of market) with a general stock index (like the Dow Jones Industrial Average DJIA). These two indices usually fluctuate in the same direction; however, technical analysts believe that the advance – decline line precedes the general financial index. In case both indices increase (decrease), the total market is said to be technically powerful (weak). Generally, technicians believe that the advance – decline line provides better indications about the total market direction than a financial index, because the latter is more influenced by large companies since it is usually based on the current value of the stock of these companies.

2. The short interest ratio

This ratio is estimated as the number of shares sold short by investors and not covered divided by the average daily trading volume. This index can be estimated for each stock separately, yet it is usually estimated for the entire market.

Investors sell short when they expect the price of a stock to fall. Thus, the higher the short interest ratio, the more the stock price will fall. However, many technicians perceive this ratio in an opposite way. A high ratio denotes potential demand for the stock by those who have already sold short and have not yet covered their sales. Note that when the ratio approaches or exceeds 3.00, there are stock price increase trends, and when it approaches or falls under 2.00, there are stock price decrease trends.

Synopsis

- **Technical analysis examines past market data to estimate future price trends. The main objective of technical analysis is to forecast the short-term price stock fluctuations and not the exact price.**
- **Technical analysis is based on the assumption that stock prices tend to move in trends which persist for long lengths of time. This happens because information, which alters supply and demand, does not come to the market at one point in time but, rather, enters the market over a period of time.**
- **Technical analysts use many different technical trading rules. However, the problem is that these rules may lead to more than one explanations. The most important technical trading rules are included in one of the following four categories: a) Contrary opinion rules, b) Follow the smart money, c) Market sentiment techniques and d) Stock price and volume techniques. In this chapter, we briefly examine the first three technical rules.**

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