Advanced Quantitative Methods for Managers

VOLUME 1

Advanced Quantitative Analysis
Note

The figures that have been included in this volume are used strictly for educational purposes and take the place of visual materials that would be presented during a lecture. They are provided only for personal use by students of the Hellenic Open University (HOU), and are accompanied by a reference to their source and/or the person who created them. The figures have been reproduced at a size that facilitates comprehension of the words and symbols in them, as well as their content in general.

Reprinting or any other form of reproduction of this volume is prohibited. The volume is intended for the purposes of teaching and examination of HOU students. It is distributed free of charge only to those who created the teaching materials, to students enrolled at HOU, and to the relevant teaching personnel; it is not available for purchase.
ADVANCED QUANTITATIVE METHODS
FOR MANAGERS

Advanced Quantitative Analysis

Note
The Hellenic Open University is responsible for the editing of this publication and the development of the text in accordance with the Methodology of Distance Learning. The scientific accuracy and completeness of the written materials are the exclusive responsibility of the authors, scientific reviewers, and academic supervisors who undertook this project.
PREPARATION OF THE TEACHING MATERIAL
of the Volume

Advanced Quantitative Analysis

Academic Supervisor for the Development of the Program and the Textbooks
George Agiomirgianakis

Author
Manolis G. Kavussanos

Scientific Reviewer
Antonios Demos

Supervision of the Methodology of Distance Learning
Antonia–Maria Chartofylaka

Language Editing
Zannis Res

Artistic Design / Artistic Layout
opusMAGNUM

Layout / Production
opusMAGNUM

Coordination of the development of the educational material
and overall supervision of the publications
HOU PROJECT TEAM /1997–2005

ISBN: 960-538-584-8

In accordance with Law 2121/1993,
the partial or total republishing, or reproduction
by any means, of this book is prohibited
without the permission of the publisher.
# CONTENTS

<table>
<thead>
<tr>
<th>Preface</th>
<th>13</th>
</tr>
</thead>
</table>

## CHAPTER 1

### Data Collection and Presentation

<table>
<thead>
<tr>
<th>The Scope of the Chapter</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Objectives</td>
<td>15</td>
</tr>
<tr>
<td>Key Words</td>
<td>15</td>
</tr>
<tr>
<td>Introductory Comments</td>
<td>15</td>
</tr>
<tr>
<td>1.1 Introduction to data sources and data collection methods</td>
<td>17</td>
</tr>
<tr>
<td>1.1.1 Primary data collection and survey methods</td>
<td>17</td>
</tr>
<tr>
<td>1.1.2 Secondary data and sources</td>
<td>18</td>
</tr>
<tr>
<td>1.1.3 Sampling methods</td>
<td>19</td>
</tr>
<tr>
<td>1.2 Presentation of data</td>
<td>23</td>
</tr>
<tr>
<td>1.2.1 Tabular (in table form) presentation of data</td>
<td>23</td>
</tr>
<tr>
<td>1.2.2 Visual presentation of data</td>
<td>29</td>
</tr>
<tr>
<td>Synopsis - Conclusions</td>
<td>39</td>
</tr>
<tr>
<td>Appendix</td>
<td>40</td>
</tr>
</tbody>
</table>

## CHAPTER 2

### Descriptive Statistics

<table>
<thead>
<tr>
<th>The Scope of the Chapter</th>
<th>43</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Objectives</td>
<td>43</td>
</tr>
<tr>
<td>Key Words</td>
<td>43</td>
</tr>
<tr>
<td>Introductory Comments</td>
<td>43</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>45</td>
</tr>
<tr>
<td>2.2 Measures of Central Tendency</td>
<td>46</td>
</tr>
<tr>
<td>2.2.1 Arithmetic mean</td>
<td>46</td>
</tr>
<tr>
<td>2.2.2 Median</td>
<td>46</td>
</tr>
<tr>
<td>2.2.3 Mode</td>
<td>47</td>
</tr>
</tbody>
</table>
2.2.4 Geometric Mean ................................................................. 47
2.2.5 The choice of average ......................................................... 49

2.3 Measures of Dispersion .......................................................... 51
2.3.1 Range .............................................................................. 51
2.3.2 Quartiles and Deciles of Datasets ....................................... 51
2.3.3 Interquartile Range ............................................................ 54
2.3.4 Quartile deviation ............................................................. 54
2.3.5 Average deviation ............................................................. 55
2.3.6 Variance .......................................................................... 55
2.3.7 Standard deviation ............................................................ 55
2.3.8 Coefficient of Variation ..................................................... 56

2.4 Shape of frequency distributions (Higher moments) ............... 58
2.4.1 Coefficient of Skewness ..................................................... 58
2.4.2 Coefficient of Kurtosis ....................................................... 58

Synopsis - Conclusions ................................................................. 62
Appendix ....................................................................................... 63

CHAPTER 3

Introduction to Probability ......................................................... 67
The Scope of the Chapter ........................................................... 67
Learning Objectives ..................................................................... 67
Key Words .................................................................................... 67
Introductory Comments ............................................................. 67

3.1 Basic Concepts .................................................................... 68
3.2 Three notions of probability .................................................. 69
3.2.1 Classical or a priori probability .......................................... 69
3.2.2 Relative frequency or empirical probability ....................... 69
3.2.3 Subjective or Bayesian probability .................................... 70

3.3 Axioms of probability ......................................................... 71

3.4 Relationships between events (multiple events) .................... 72
3.4.1 Mutually exclusive events .............................................. 72
3.4.2 Not mutually exclusive events ......................................... 74
3.4.3 Independent events ......................................................... 74
3.4.4 Dependent events .......................................................... 75

3.5 Subjective probability and Bayes’ theorem ............................ 77

3.6 Examples on probability trees .............................................. 78

3.7 Introduction to probability distributions .............................. 82
Synopsis - Conclusions ............................................................... 83
Appendix ....................................................................................... 84
CHAPTER 6
Statistical Inference:
Estimation, Sampling, Confidence Intervals 133

The Scope of the Chapter ................................................................. 133
Learning Objectives ......................................................................... 133
Key Words .......................................................................................... 133
Introductory Comments ................................................................. 133
6.1 Definitions - Concepts ............................................................. 135
6.2 The sampling distribution of the sample mean ....................... 137
6.3 Estimators .................................................................................. 142
   6.3.1 Introduction ......................................................................... 142
   6.3.2 Choosing between estimators .............................................. 143
   6.3.3 Properties of point estimators for \( \mu \) and \( \sigma^2 \) ............... 144
6.4 Confidence intervals for the population mean ......................... 145
6.5 Population proportion, \( \pi \) ......................................................... 151
6.6 Determining the optimal sample size, \( n \) ................................. 153
6.7 Confidence intervals for the difference of means and percentages 156
   6.7.1 Confidence intervals for the difference of population means ...... 156
   6.7.2 Confidence interval for the difference of population proportions ........ 158
6.8 Inferences about the population variance ................................. 160
Synopsis - Conclusions ................................................................. 161
Appendix .......................................................................................... 162

CHAPTER 7
Statistical Inference: Hypothesis Testing 167

The Scope of the Chapter ................................................................. 167
Learning Objectives ......................................................................... 167
Key Words .......................................................................................... 167
Introductory Comments ................................................................. 167
7.1 Introduction, Definition - Concepts ....................................... 169
7.2 Hypothesis testing about the population mean or proportion .... 172
7.3 Hypothesis testing about differences between two means or proportions ....................................................... 176
7.4 Hypothesis testing about the population variance .................... 179
7.5 Hypothesis testing about the equality of variances .................. 182
7.6 Hypothesis testing about values of correlation coefficients ........ 185
7.7 Contingency tables and goodness of fit tests .......................... 187
7.7.1 Testing the independence of two categorical variables................. 187
7.7.2 Testing whether observed frequencies differ from expected
frequencies when more than two outcomes are possible............... 190
7.7.3 Testing the nature of the sampling distribution ......................... 191
7.7.4 Testing whether more than two proportions are equal................. 192

Synopsis - Conclusions ..................................................................... 194
Appendix .......................................................................................... 195

CHAPTER 8

Multivariate Analysis and Regression Models 201

The Scope of the Chapter................................................................. 201
Learning Objectives ....................................................................... 201
Key Words....................................................................................... 201
Introductory Comments ............................................................... 201
8.1 Discrete probability distributions and moments of random
variables ....................................................................................... 202
8.1.1 Two-variable or bivariate probability distributions ................. 202
8.1.2 Moments of bivariate random variables............................... 204
8.2 Conditioning of variables and regression analysis..................... 209
8.2.1 Introduction ........................................................................... 209
8.2.2 Deriving the OLS (Ordinary Least Squares) estimators $b_1, b_2$..... 211
8.2.3 Properties of the fitted OLS line ........................................... 212
8.2.4 Properties of the OLS estimators ........................................... 213
8.2.5 The second moments of the estimators $b_1$ and $b_2$.................. 214
8.2.6 The problem of statistical inference ....................................... 215
8.2.7 Goodness of Fit: $R^2$ - The coefficient of determination .......... 218
8.2.8 Analysis of Variance (ANOVA) ............................................. 220
8.2.9 Forecasting of $Y$ and forecast intervals................................. 222
8.3 Extension of results to multivariate regression ......................... 230
Synopsis - Conclusions ................................................................. 233
Appendix ....................................................................................... 234

Bibliography ................................................................................. 235

Glossary ......................................................................................... 237

Statistical Tables ........................................................................... 249