Forecasting Exchange Rates:
A comparison of naïve, linear, and regime-switching models

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Abstract
This paper uses monthly data on euro exchange rates vis-a-vis the pound, the dollar and the yen, covering the period 1999-2010, to compare the forecasting ability of alternative stochastic exchange rate representations. In particular, we test the out-of-sample forecasting performance of a random walk, a vector autoregressive representation reflecting the dynamics of linear structural exchange rate models, and a non-linear Markov switching regimes process. These statistical models are evaluated in terms of the root mean square error of one-month to twelve-month out-of-sample forecasts. The empirical evidence points to the random walk puzzle, that is, the superiority of the naïve model in forecasting exchange rates over short horizons. However, it seems that structural approaches provide a better setting for predicting euro rates over longer periods.

JEL classification: F31, F37

Keywords: Forecasting, random walk, linear models, switching regimes.

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1. Introduction

The superiority of a random walk in forecasting exchange rates out of sample (Meese and Rogoff, 1983) gave rise to extensive research dealing with the empirical validity of the building blocks of exchange rate theories and the econometric methods used in testing structural models. This research project has come to a rather general consensus that exchange rate asset market models are not supported empirically and, in any case, they cannot improve on random walk forecasts.

However, the finding of Meese and Rogoff (1983) that the random walk is also a better predictor than other univariate time series models has been challenged by Engel and Hamilton (1990) who reported evidence in favour of a Markov switching regimes process for exchange rate changes. Indeed, these authors found that a model of stochastic segmented trends produces better in-sample and out-of-sample exchange rate forecasts on the basis of mean square error. Also, Kirikos (2000), using a much larger data set, reported that a random walk gives consistently better in-sample forecasts but the Markov switching regimes model predicts better for short out-of-sample horizons when the post-sample period is narrowed towards the end of the full sample.

In this paper, we re-examine the forecasting performance of a Markov switching process relative to those of a random walk and of a vector autoregression (VAR) using a data set on euro rates. In particular, the three models are compared by means of the root mean square error (RMSE) of forecasts based on monthly data for the currencies of the UK, USA and Japan relative to the euro (€) over the period 1999 - 2010. The empirical evidence points to the random walk puzzle, that is, the superiority of the naïve model in forecasting exchange rates over short horizons. However, it seems that
structural approaches provide a better setting for predicting euro rates over longer periods.

The forecasting methodology is discussed in the next section and the results are presented in Section 3. The fourth section contains a summary and conclusions.

2. Methodology

Let $e_t$ ($t = 1, 2, ..., T$) be the natural logarithm of the exchange rate and $s_t$ the first difference of $e_t$ (i.e. $s_t = e_t - e_{t-1}$). Then, the $k$-period-ahead forecast of a random walk with a drift parameter is given by:

$$
\hat{e}_{t+k|t} = e_t + k \cdot \bar{s}
$$

(1)

where $\hat{e}_{t+k|t}$ is the forecast of $e_{t+k}$ based on information at time $t$,

$$
\bar{s} = \frac{1}{n-1} \sum_{t=1}^{n-1} s_t
$$

is the sample mean of $s_t$, and $n$ is any sub-sample ($n \leq T$) on which out-of-sample forecasts are based.

An alternative process is the following:

$$
s_t = \mu_{h_t} + u_t, \quad u_t \sim N(0, \sigma_{h_t}^2)
$$

(2)

where $h_t$ is an unobserved state variable that takes on values in the set \{1, 2\}, and $u$ is an error term. State 1 will be referred to as the depreciation state while state 2 will be associated with an appreciation of the relevant currency relative to the euro. Thus, Equation 2 allows for different means and variances across regimes. The state variable $h_t$ is assumed to follow an irreducible Markov chain with stationary transition probability matrix:

$$
P = \begin{bmatrix}
    p_{11} & p_{12} \\
    p_{21} & p_{22}
\end{bmatrix}
$$

(3)
where \( p_{ij} = Pr(h_t = j | h_{t-1} = i) \), \( i, j = 1, 2 \). Under this assumption the \( k \)-period-ahead forecast of \( s_{t+k} \), based on time-\( t \) information is (Hamilton, 1993; Kirikos, 1996):

\[
\tilde{s}_{t+k|t} = E(s_{t+k} | S_t) = \alpha'_t \cdot P^k \cdot \mu_h
\]

where \( S_t \) is the history of \( s \) up to time \( t \), \( \alpha'_t = \left[ Pr(h_t = 1 | S_t) \quad Pr(h_t = 2 | S_t) \right] \) is the vector of probabilistic inferences about the state at date \( t \) (Hamilton, 1990, 1993), and \( \mu_h' = [\mu_1 \quad \mu_2] \) is the vector of state means. It should be noted that forecasts given by Equation 4 are nonlinear since the inferences \( \alpha'_t \) are produced by a nonlinear filter.

Maximum likelihood estimates of the means \( (\mu_1, \mu_2) \), variances \( (\sigma_1^2, \sigma_2^2) \), and the transition probabilities \( (p_{11}, p_{22}) \) can be obtained through the EM algorithm which is a method of maximizing the sample likelihood function by iterating on the normal equations (Hamilton, 1990).

Given Equation 4, forecasts of the logarithm of the exchange rate are computed by:

\[
\tilde{e}_{t+k|t} = e_t + \tilde{s}_{t+1|t} + \tilde{s}_{t+2|t} + \ldots + \tilde{s}_{t+k|t}
\]

Next, we look at a class of linear forecasts along the lines of structural asset market models of the exchange rate. In particular, we consider vector autoregressive (VAR) representations for exchange rates and observed fundamentals as proposed by Engel and West (2004, 2005), that is, VARs in the exchange rate and the variables \( y_t - y_t^*, p_t - p_t^*, i_t - i_t^* \), where \( y_t \) is the logarithm of domestic GDP, \( p_t \) is the logarithm of the domestic price level, \( i_t \) is the domestic interest rate and starred variables are the foreign counterparts.
Post-sample forecasts are computed through rolling estimation of the models. That is, initial $k$-period-ahead forecasts are based on parameter estimates obtained with a sub-sample of size $n$. Then, the models are re-estimated by adding the next available observation and new forecasts are generated. This procedure continues until the subsample size becomes $T-k$, where $T$ is the full sample size.

The forecasting accuracy of the models is measured by the root mean square error (RMSE) of forecasts:

$$RMSE = \left[ \frac{1}{T-n-k+1} \sum_{i=0}^{T-n-k} (\hat{y}_{n+i+k|n+i} - y_{n+i+k})^2 \right]^{1/2}$$

(6)

where $n$ is the initial sub-sample size and $k$ is the forecast horizon.

### 3. Empirical Results

The forecasting performance of the models is compared on monthly data on the currencies of the UK (£), USA ($), and Japan (¥) vis-à-vis the euro (€) covering the period from January 1999 to March 2010 (135 observations).\(^1\)

A caveat is in order with regard to the estimates of the VAR model. Our preliminary analysis shows that variables included in the VAR are difference stationary, except for the interest differentials between the eurozone and Japan as well as between the eurozone and the UK. Thus, the first differences of the variables with a unit root are included in the VAR and lag lengths have arbitrarily been limited to 2. This approach does not rule out the possibility of over differencing the system as there might be cointegrating relationships among the information variables and, in this case, an error-correction representation would be more appropriate. Tests for cointegration have not

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\(^1\) Data and sources are described in the appendix.
been conducted in this version and, therefore, the results reported below with regard to the VAR should be seen as preliminary and interpreted with caution.

The RMSEs of out-of-sample forecasts at horizons of 1 to 12 months are reported in Table 1 for the post-sample period 2008:4 – 2010:3 and show that the random walk model, with or without drift, performs better than the Markov process for all currencies and forecast horizons.

However, the relative forecasting performance of the VAR for the €/£ rate improves drastically with the forecast horizon and beats the random walk at horizons of 3 or more months. The same is true for the €/$ rate even though the relative improvement here is not as fast and overthrows the superiority of the naïve model only at 12-month-ahead forecasts. Finally the forecasting superiority of the random walk appears to be overwhelming in the case of the €/¥ rate.

Table 1. **RMSE of out-of-sample forecasts**

<table>
<thead>
<tr>
<th></th>
<th>Forecast Horizon (months)</th>
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<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>12</td>
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<td>2008:4 – 2010:3</td>
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<tr>
<td>Random walk with</td>
<td>C/E</td>
<td>0.0263</td>
<td>0.0476</td>
<td>0.0714</td>
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<tr>
<td>Random walk without</td>
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<td>0.0243</td>
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<tr>
<td>Markov model</td>
<td>C/E</td>
<td>0.0212</td>
<td>0.0638</td>
<td>0.1088</td>
<td>0.1663</td>
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<tr>
<td>VAR</td>
<td>C/E</td>
<td>0.0377</td>
<td>0.0379</td>
<td>0.0373</td>
<td>0.0362</td>
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<td>Random walk with</td>
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<td>0.0096</td>
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<tr>
<td>Random walk without</td>
<td>C/$</td>
<td>0.0107</td>
<td>0.0560</td>
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<tr>
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<tr>
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<td>VAR</td>
<td>C/¥</td>
<td>3.6127</td>
<td>3.6358</td>
<td>3.6715</td>
<td>3.5583</td>
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</table>
4. Conclusions

Based on a data set for euro rates over the period 1999-2010, we obtained evidence that the out-of-sample forecasting performance of a random walk is superior to that of a Markov switching regimes model. This suggests that policy response explanations associated with non-linearities in euro rates (see Kirikos, 2002) may not be relevant. However, fundamentals along the lines of structural asset market models seem to have an impact over longer periods since a linear VAR model in exchange rates and fundamentals produces better 12-month-ahead forecasts for at least two euro rates. Thus, our results reproduce the random walk puzzle in short-run forecasting but point to the importance of fundamentals over longer periods.

This work could be usefully extended by examining the robustness of results when the post-sample period is varied as well as by improving the specification for exchange rates and asset market fundamentals through a more thorough investigation of their time series properties, in particular cointegrating relationships which may suggest an error-correction representation.

Acknowledgement

Thanks are due to Ms. Foteini Taxaki for collecting and providing the data used in this work.
Appendix

Data description and sources

The data set covers the period from January 1999 to March 2010 (135 monthly observations). The eurozone is considered to be the home country and the performance of the models is assessed for the currencies of Japan, the UK, and the USA against the euro.

The spot exchange rate series were taken from the OECD site. Interest rates are yields on annual treasury bills (T-bills). The European T-bill data was taken from the European Central Bank (ECB) site, the Japanese T-bill data was taken from the web site of the ministry of Finance of Japan, while American T-bill and UK T-bill data was obtained from the Federal Reserve Bank and the Bank of England web sites, respectively.

Data on the consumer price index and the GDP, which was proxied by the industrial production index, was obtained from OECD and the ECB.
References


